Chapter 7

Energy and Energy Transfer

▲ *On a wind farm, the moving air does work on the blades of the windmills, causing the blades and the rotor of an electrical generator to rotate. Energy is transferred out of the system of the windmill by means of electricity. (Billy Hustace/Getty Images)*

CHAPTER OUTLINE

The concept of energy is one of the most important topics in science and engineering. In everyday life, we think of energy in terms of fuel for transportation and heating, electricity for lights and appliances, and foods for consumption. However, these ideas do not really define energy. They merely tell us that fuels are needed to do a job and that those fuels provide us with something we call energy.

The definitions of quantities such as position, velocity, acceleration, and force and associated principles such as Newton's second law have allowed us to solve a variety of problems. Some problems that could theoretically be solved with Newton's laws, however, are very difficult in practice. These problems can be made much simpler with a different approach. In this and the following chapters, we will investigate this new approach, which will include definitions of quantities that may not be familiar to you. Other quantities may sound familiar, but they may have more specific meanings in physics than in everyday life. We begin this discussion by exploring the notion of energy.

Energy is present in the Universe in various forms. *Every* physical process that occurs in the Universe involves energy and energy transfers or transformations. Unfortunately, despite its extreme importance, energy cannot be easily defined. The variables in previous chapters were relatively concrete; we have everyday experience with velocities and forces, for example. The notion of energy is more abstract, although we do have *experiences* with energy, such as running out of gasoline, or losing our electrical service if we forget to pay the utility bill.

The concept of energy can be applied to the dynamics of a mechanical system without resorting to Newton's laws. This "energy approach" to describing motion is especially useful when the force acting on a particle is not constant; in such a case, the acceleration is not constant, and we cannot apply the constant acceleration equations that were developed in Chapter 2. Particles in nature are often subject to forces that vary with the particles' positions. These forces include gravitational forces and the force exerted on an object attached to a spring. We shall describe techniques for treating such situations with the help of an important concept called *conservation of energy*. This approach extends well beyond physics, and can be applied to biological organisms, technological systems, and engineering situations.

Our problem-solving techniques presented in earlier chapters were based on the motion of a particle or an object that could be modeled as a particle. This was called the *particle model*. We begin our new approach by focusing our attention on a *system* and developing techniques to be used in a *system model*.

7.1 Systems and Environments

In the system model mentioned above, we focus our attention on a small portion of the Universe—the **system**—and ignore details of the rest of the Universe outside of the system. A critical skill in applying the system model to problems is *identifying the system*.

A valid system may

- be a single object or particle
- be a collection of objects or particles
- be a region of space (such as the interior of an automobile engine combustion cylinder)
- vary in size and shape (such as a rubber ball, which deforms upon striking a wall)

Identifying the *need* for a system approach to solving a problem (as opposed to a particle approach) is part of the "categorize" step in the General Problem-Solving Strategy outlined in Chapter 2. Identifying the particular system and its nature is part of the "analyze" step.

No matter what the particular system is in a given problem, there is a **system boundary,** an imaginary surface (not necessarily coinciding with a physical surface) that divides the Universe into the system and the **environment** surrounding the system.

As an example, imagine a force applied to an object in empty space. We can define the object as the system. The force applied to it is an influence on the system from the environment that acts across the system boundary. We will see how to analyze this situation from a system approach in a subsequent section of this chapter.

Another example is seen in Example 5.10 (page 130). Here the system can be defined as the combination of the ball, the cube, and the string. The influence from the environment includes the gravitational forces on the ball and the cube, the normal and friction forces on the cube, and the force exerted by the pulley on the string. The forces exerted by the string on the ball and the cube are internal to the system and, therefore, are not included as an influence from the environment.

We shall find that there are a number of mechanisms by which a system can be influenced by its environment. The first of these that we shall investigate is *work.*

PITFALL PREVENTION

7.1 Identify the System

The most important step to take in solving a problem using the energy approach is to identify the appropriate system of interest. Make sure this is the *first* step you take in solving a problem.

7.2 Work Done by a Constant Force

Almost all the terms we have used thus far—velocity, acceleration, force, and so on—convey a similar meaning in physics as they do in everyday life. Now, however, we encounter a term whose meaning in physics is distinctly different from its everyday meaning—*work.*

To understand what work means to the physicist, consider the situation illustrated in Figure 7.1. A force is applied to a chalkboard eraser, and the eraser slides

Charles D. Winters

(a) (b) (c)

Figure 7.1 An eraser being pushed along a chalkboard tray.

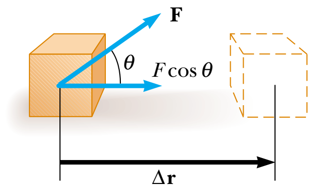


Figure 7.2 If an object undergoes a displacement $\Delta\mathbf{r}$ under the action of a constant force $\mathbf{F}$, the work done by the force is $F\Delta r\cos\theta$.

along the tray. If we want to know how effective the force is in moving the eraser, we must consider not only the magnitude of the force but also its direction. Assuming that the magnitude of the applied force is the same in all three photographs, the push applied in Figure 7.1b does more to move the eraser than the push in Figure 7.1a. On the other hand, Figure 7.1c shows a situation in which the applied force does not move the eraser at all, regardless of how hard it is pushed. (Unless, of course, we apply a force so great that we break the chalkboard tray.) So, in analyzing forces to determine the work they do, we must consider the vector nature of forces. We must also know how far the eraser moves along the tray if we want to determine the work associated with that displacement. Moving the eraser 3 m requires more work than moving it 2 cm.

Let us examine the situation in Figure 7.2, where an object undergoes a displacement along a straight line while acted on by a constant force $\mathbf{F}$ that makes an angle θ with the direction of the displacement.

Work done by a constant force

The **work** W done on a system by an agent exerting a constant force on the system is the product of the magnitude F of the force, the magnitude Δr of the displacement of the point of application of the force, and $\cos\theta$, where θ is the angle between the force and displacement vectors:

$$W \equiv F\Delta r\cos\theta \tag{7.1}$$

As an example of the distinction between this definition of work and our everyday understanding of the word, consider holding a heavy chair at arm's length for 3 min. At the end of this time interval, your tired arms may lead you to think that you have done a considerable amount of work on the chair. According to our definition, however, you have done no work on it whatsoever.[1] You exert a force to support the chair, but you do not move it. A force does no work on an object if the force does not move through a displacement. This can be seen by noting that if $\Delta r = 0$, Equation 7.1 gives $W = 0$—the situation depicted in Figure 7.1c.

Also note from Equation 7.1 that the work done by a force on a moving object is zero when the force applied is perpendicular to the displacement of its point of

PITFALL PREVENTION

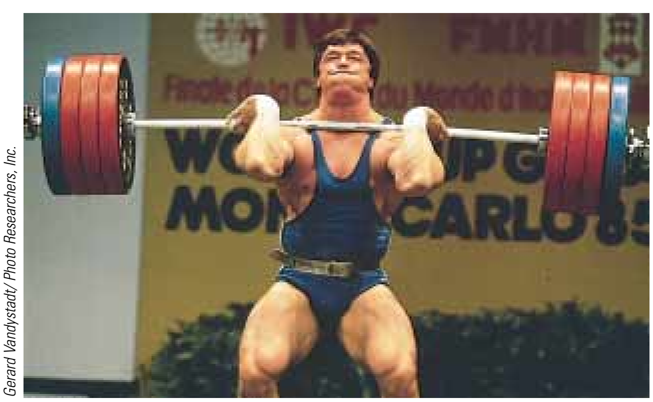
7.2 What is being Displaced?

The displacement in Equation 7.1 is that of *the point of application of the force.* If the force is applied to a particle or a non-deformable, non-rotating system, this displacement is the same as the displacement of the particle or system. For deformable systems, however, these two displacements are often not the same.

PITFALL PREVENTION

7.3 Work is Done by . . . on . . .

Not only must you identify the system, you must also identify the interaction of the system with the environment. When discussing work, always use the phrase, "the work done by . . . on . . . " After "by," insert the part of the environment that is interacting directly with the system. After "on," insert the system. For example, "the work done by the hammer on the nail" identifies the nail as the system and the force from the hammer represents the interaction with the environment. This is similar to our use in Chapter 5 of "the force exerted by . . . on . . ."



Gerard Vandystadt/Photo Researchers, Inc.

The weightlifter does no work on the weights as he holds them on his shoulders. (If he could rest the bar on his shoulders and lock his knees, he would be able to support the weights for quite some time.) Did he do any work when he raised the weights to this height?

[1] Actually, you do work while holding the chair at arm's length because your muscles are continuously contracting and relaxing; this means that they are exerting internal forces on your arm. Thus, work is being done by your body—but internally on itself rather than on the chair.

application. That is, if $\theta = 90°$, then $W = 0$ because $\cos 90° = 0$. For example, in Figure 7.3, the work done by the normal force on the object and the work done by the gravitational force on the object are both zero because both forces are perpendicular to the displacement and have zero components along an axis in the direction of $\Delta\mathbf{r}$.

The sign of the work also depends on the direction of $\mathbf{F}$ relative to $\Delta\mathbf{r}$. The work done by the applied force is positive when the projection of $\mathbf{F}$ onto $\Delta\mathbf{r}$ is in the same direction as the displacement. For example, when an object is lifted, the work done by the applied force is positive because the direction of that force is upward, in the same direction as the displacement of its point of application. When the projection of $\mathbf{F}$ onto $\Delta\mathbf{r}$ is in the direction opposite the displacement, W is negative. For example, as an object is lifted, the work done by the gravitational force on the object is negative. The factor $\cos\theta$ in the definition of W (Eq. 7.1) automatically takes care of the sign.

If an applied force $\mathbf{F}$ is in the same direction as the displacement $\Delta\mathbf{r}$, then $\theta = 0$ and $\cos 0 = 1$. In this case, Equation 7.1 gives

$$W = F\Delta r$$

Work is a scalar quantity, and its units are force multiplied by length. Therefore, the SI unit of work is the **newton · meter** (N · m). This combination of units is used so frequently that it has been given a name of its own: the **joule** (J).

An important consideration for a system approach to problems is to note that **work is an energy transfer.** If W is the work done on a system and W is positive, energy is transferred *to* the system; if W is negative, energy is transferred *from* the system. Thus, if a system interacts with its environment, this interaction can be described as a transfer of energy across the system boundary. This will result in a change in the energy stored in the system. We will learn about the first type of energy storage in Section 7.5, after we investigate more aspects of work.

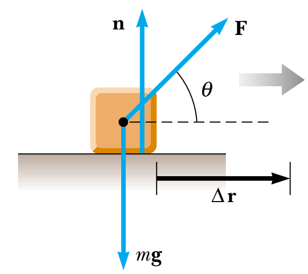


Figure 7.3 When an object is displaced on a frictionless, horizontal surface, the normal force **n** and the gravitational force $m\mathbf{g}$ do no work on the object. In the situation shown here, **F** is the only force doing work on the object.

PITFALL PREVENTION

7.4 Cause of the Displacement

We can calculate the work done by a force on an object, but that force is *not* necessarily the cause of the object's displacement. For example, if you lift an object, work is done by the gravitational force, although gravity is not the cause of the object moving upward!

Quick Quiz 7.1 The gravitational force exerted by the Sun on the Earth holds the Earth in an orbit around the Sun. Let us assume that the orbit is perfectly circular. The work done by this gravitational force during a short time interval in which the Earth moves through a displacement in its orbital path is (a) zero (b) positive (c) negative (d) impossible to determine.

Quick Quiz 7.2 Figure 7.4 shows four situations in which a force is applied to an object. In all four cases, the force has the same magnitude, and the displacement of the object is to the right and of the same magnitude. Rank the situations in order of the work done by the force on the object, from most positive to most negative.

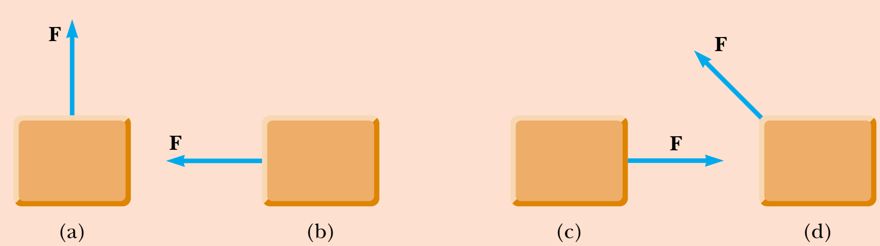


Figure 7.4 (Quick Quiz 7.2)

Example 7.1 Mr. Clean

A man cleaning a floor pulls a vacuum cleaner with a force of magnitude $F = 50.0$ N at an angle of 30.0° with the horizontal (Fig. 7.5a). Calculate the work done by the force on the vacuum cleaner as the vacuum cleaner is displaced 3.00 m to the right.

Solution Figure 7.5a helps conceptualize the situation. We are given a force, a displacement, and the angle between the two vectors, so we can categorize this as a simple problem that will need minimal analysis. To analyze the situation, we identify the vacuum cleaner as the system and draw a free-body diagram as shown in Figure 7.5b. Using the definition of work (Eq. 7.1),

$$W = F\,\Delta r\cos\theta = (50.0\text{ N})(3.00\text{ m})(\cos 30.0°)$$

$$= 130\,\text{N}\cdot\text{m} = \boxed{130\text{ J}}$$

To finalize this problem, notice in this situation that the normal force $\mathbf{n}$ and the gravitational $\mathbf{F}_g = m\mathbf{g}$ do no work on the vacuum cleaner because these forces are perpendicular to its displacement.

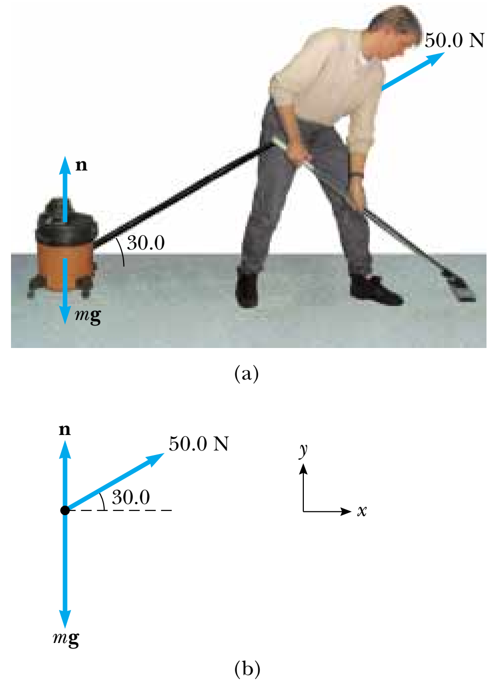


Figure 7.5 (Example 7.1) (a) A vacuum cleaner being pulled at an angle of 30.0° from the horizontal. (b) Free-body diagram of the forces acting on the vacuum cleaner.

7.3 The Scalar Product of Two Vectors

Because of the way the force and displacement vectors are combined in Equation 7.1, it is helpful to use a convenient mathematical tool called the **scalar product** of two vectors. We write this scalar product of vectors $\mathbf{A}$ and $\mathbf{B}$ as $\mathbf{A}\cdot\mathbf{B}$. (Because of the dot symbol, the scalar product is often called the **dot product.**)

In general, the scalar product of any two vectors $\mathbf{A}$ and $\mathbf{B}$ is a scalar quantity equal to the product of the magnitudes of the two vectors and the cosine of the angle θ between them:

Scalar product of any two vectors A and B

$$\mathbf{A}\cdot\mathbf{B} \equiv AB\cos\theta \tag{7.2}$$

Note that $\mathbf{A}$ and $\mathbf{B}$ need not have the same units, as is the case with any multiplication.

Comparing this definition to Equation 7.1, we see that we can express Equation 7.1 as a scalar product:

$$W = F\,\Delta r\cos\theta = \mathbf{F}\cdot\Delta\mathbf{r} \tag{7.3}$$

In other words, $\mathbf{F}\cdot\Delta\mathbf{r}$ (read "$\mathbf{F}$ dot $\Delta\mathbf{r}$") is a shorthand notation for $F\,\Delta r\cos\theta$.

Before continuing with our discussion of work, let us investigate some properties of the dot product. Figure 7.6 shows two vectors $\mathbf{A}$ and $\mathbf{B}$ and the angle θ between them that is used in the definition of the dot product. In Figure 7.6, $B\cos\theta$ is the projection of $\mathbf{B}$ onto $\mathbf{A}$. Therefore, Equation 7.2 means that $\mathbf{A}\cdot\mathbf{B}$ is the product of the magnitude of $\mathbf{A}$ and the projection of $\mathbf{B}$ onto $\mathbf{A}$.[2]

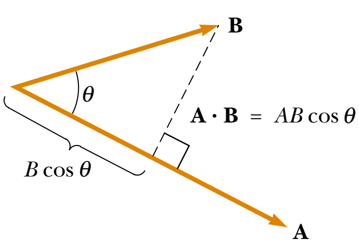


Figure 7.6 The scalar product $\mathbf{A}\cdot\mathbf{B}$ equals the magnitude of $\mathbf{A}$ multiplied by $B\cos\theta$, which is the projection of $\mathbf{B}$ onto $\mathbf{A}$.

[2] This is equivalent to stating that $\mathbf{A}\cdot\mathbf{B}$ equals the product of the magnitude of $\mathbf{B}$ and the projection of $\mathbf{A}$ onto $\mathbf{B}$.

From the right-hand side of Equation 7.2 we also see that the scalar product is **commutative**.[3] That is,

$$\mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A}$$

Finally, the scalar product obeys the **distributive law of multiplication,** so that

$$\mathbf{A}\cdot(\mathbf{B} + \mathbf{C}) = \mathbf{A}\cdot\mathbf{B} + \mathbf{A}\cdot\mathbf{C}$$

The dot product is simple to evaluate from Equation 7.2 when $\mathbf{A}$ is either perpendicular or parallel to $\mathbf{B}$. If $\mathbf{A}$ is perpendicular to $\mathbf{B}$ ($\theta = 90°$), then $\mathbf{A}\cdot\mathbf{B} = 0$. (The equality $\mathbf{A}\cdot\mathbf{B} = 0$ also holds in the more trivial case in which either $\mathbf{A}$ or $\mathbf{B}$ is zero.) If vector $\mathbf{A}$ is parallel to vector $\mathbf{B}$ and the two point in the same direction ($\theta = 0$), then $\mathbf{A}\cdot\mathbf{B} = AB$. If vector $\mathbf{A}$ is parallel to vector $\mathbf{B}$ but the two point in opposite directions ($\theta = 180°$), then $\mathbf{A}\cdot\mathbf{B} = -AB$. The scalar product is negative when $90° < \theta \leq 180°$.

The unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$, which were defined in Chapter 3, lie in the positive x, y, and z directions, respectively, of a right-handed coordinate system. Therefore, it follows from the definition of $\mathbf{A}\cdot\mathbf{B}$ that the scalar products of these unit vectors are

Dot products of unit vectors

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = 1 \tag{7.4}$$

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{i}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = 0 \tag{7.5}$$

Equations 3.18 and 3.19 state that two vectors $\mathbf{A}$ and $\mathbf{B}$ can be expressed in component vector form as

$$\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$

$$\mathbf{B} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$$

Using the information given in Equations 7.4 and 7.5 shows that the scalar product of $\mathbf{A}$ and $\mathbf{B}$ reduces to

$$\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z \tag{7.6}$$

(Details of the derivation are left for you in Problem 6.) In the special case in which $\mathbf{A} = \mathbf{B}$, we see that

$$\mathbf{A}\cdot\mathbf{A} = A_x^2 + A_y^2 + A_z^2 = A^2$$

Quick Quiz 7.3 Which of the following statements is true about the relationship between $\mathbf{A}\cdot\mathbf{B}$ and $(-\mathbf{A})\cdot(-\mathbf{B})$? (a) $\mathbf{A}\cdot\mathbf{B} = -[(-\mathbf{A})\cdot(-\mathbf{B})]$; (b) If $\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$, then $(-\mathbf{A})\cdot(-\mathbf{B}) = AB\cos(\theta + 180°)$; (c) Both (a) and (b) are true. (d) Neither (a) nor (b) is true.

Quick Quiz 7.4 Which of the following statements is true about the relationship between the dot product of two vectors and the product of the magnitudes of the vectors? (a) $\mathbf{A}\cdot\mathbf{B}$ is larger than AB; (b) $\mathbf{A}\cdot\mathbf{B}$ is smaller than AB; (c) $\mathbf{A}\cdot\mathbf{B}$ could be larger or smaller than AB, depending on the angle between the vectors; (d) $\mathbf{A}\cdot\mathbf{B}$ could be equal to AB.

[3] This may seem obvious, but in Chapter 11 you will see another way of combining vectors that proves useful in physics and is not commutative.

PITFALL PREVENTION

7.5 Work is a Scalar

Although Equation 7.3 defines the work in terms of two vectors, *work is a scalar*—there is no direction associated with it. *All* types of energy and energy transfer are scalars. This is a major advantage of the energy approach—we don't need vector calculations!

Example 7.2 The Scalar Product

The vectors $\mathbf{A}$ and $\mathbf{B}$ are given by $\mathbf{A} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ and $\mathbf{B} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$.

(A) Determine the scalar product $\mathbf{A}\cdot\mathbf{B}$.

Solution Substituting the specific vector expressions for $\mathbf{A}$ and $\mathbf{B}$, we find,

$$\begin{aligned}\mathbf{A}\cdot\mathbf{B} &= (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})\cdot(-\hat{\mathbf{i}} + 2\hat{\mathbf{j}})\\ &= -2\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} + 2\hat{\mathbf{i}}\cdot 2\hat{\mathbf{j}} - 3\hat{\mathbf{j}}\cdot\hat{\mathbf{i}} + 3\hat{\mathbf{j}}\cdot 2\hat{\mathbf{j}}\\ &= -2(1) + 4(0) - 3(0) + 6(1)\\ &= -2 + 6 = \boxed{4}\end{aligned}$$

where we have used the facts that $\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = 1$ and $\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{i}} = 0$. The same result is obtained when we use Equation 7.6 directly, where $A_x = 2$, $A_y = 3$, $B_x = -1$, and $B_y = 2$.

(B) Find the angle θ between $\mathbf{A}$ and $\mathbf{B}$.

Solution The magnitudes of $\mathbf{A}$ and $\mathbf{B}$ are

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(2)^2 + (3)^2} = \sqrt{13}$$
$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(-1)^2 + (2)^2} = \sqrt{5}$$

Using Equation 7.2 and the result from part (a) we find that

$$\cos\theta = \frac{\mathbf{A}\cdot\mathbf{B}}{AB} = \frac{4}{\sqrt{13}\sqrt{5}} = \frac{4}{\sqrt{65}}$$

$$\theta = \cos^{-1}\frac{4}{8.06} = \boxed{60.2^\circ}$$

Example 7.3 Work Done by a Constant Force

A particle moving in the xy plane undergoes a displacement $\Delta\mathbf{r} = (2.0\hat{\mathbf{i}} + 3.0\hat{\mathbf{j}})$ m as a constant force $\mathbf{F} = (5.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}})$ N acts on the particle.

(A) Calculate the magnitudes of the displacement and the force.

Solution We use the Pythagorean theorem:

$$\Delta r = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(2.0)^2 + (3.0)^2} = \boxed{3.6\text{ m}}$$

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(5.0)^2 + (2.0)^2} = \boxed{5.4\text{ N}}$$

(B) Calculate the work done by $\mathbf{F}$.

Solution Substituting the expressions for $\mathbf{F}$ and $\Delta\mathbf{r}$ into Equation 7.3 and using Equations 7.4 and 7.5, we obtain

$$\begin{aligned}W &= \mathbf{F}\cdot\Delta\mathbf{r} = [(5.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}})\text{ N}]\cdot[(2.0\hat{\mathbf{i}} + 3.0\hat{\mathbf{j}})\text{ m}]\\ &= (5.0\hat{\mathbf{i}}\cdot 2.0\hat{\mathbf{i}} + 5.0\hat{\mathbf{i}}\cdot 3.0\hat{\mathbf{j}} + 2.0\hat{\mathbf{j}}\cdot 2.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}}\cdot 3.0\hat{\mathbf{j}})\text{N}\cdot\text{m}\\ &= [10 + 0 + 0 + 6]\text{ N}\cdot\text{m} = \boxed{16\text{ J}}\end{aligned}$$

7.4 Work Done by a Varying Force

Consider a particle being displaced along the x axis under the action of a force that varies with position. The particle is displaced in the direction of increasing x from $x = x_i$ to $x = x_f$. In such a situation, we cannot use $W = F\Delta r\cos\theta$ to calculate the work done by the force because this relationship applies only when $\mathbf{F}$ is constant in magnitude and direction. However, if we imagine that the particle undergoes a very small displacement Δx, shown in Figure 7.7a, the x component F_x of the force is approximately constant over this small interval; for this small displacement, we can approximate the work done by the force as

$$W \approx F_x\Delta x$$

This is just the area of the shaded rectangle in Figure 7.7a. If we imagine that the F_x versus x curve is divided into a large number of such intervals, the total work done for the displacement from x_i to x_f is approximately equal to the sum of a large number of such terms:

$$W \approx \sum_{x_i}^{x_f} F_x\Delta x$$

If the size of the displacements is allowed to approach zero, the number of terms in the sum increases without limit but the value of the sum approaches a definite value equal to the area bounded by the F_x curve and the x axis:

$$\lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x \, dx$$

Therefore, we can express the work done by F_x as the particle moves from x_i to x_f as

$$W = \int_{x_i}^{x_f} F_x \, dx \qquad (7.7)$$

This equation reduces to Equation 7.1 when the component $F_x = F \cos \theta$ is constant.

If more than one force acts on a system *and the system can be modeled as a particle*, the total work done on the system is just the work done by the net force. If we express the net force in the x direction as ΣF_x, then the total work, or *net work*, done as the particle moves from x_i to x_f is

$$\sum W = W_{\text{net}} = \int_{x_i}^{x_f} \left(\sum F_x \right) dx \qquad (7.8)$$

If the system cannot be modeled as a particle (for example, if the system consists of multiple particles that can move with respect to each other), we cannot use Equation 7.8. This is because different forces on the system may move through different displacements. In this case, we must evaluate the work done by each force separately and then add the works algebraically.

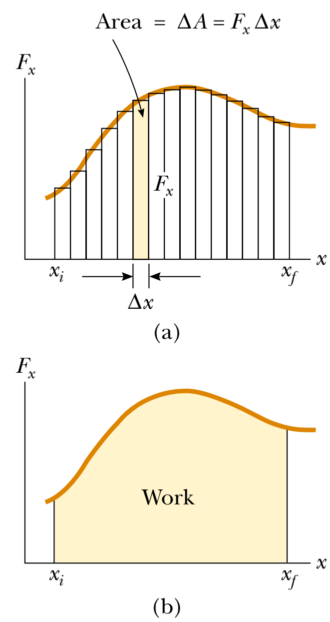


Figure 7.7 (a) The work done by the force component F_x for the small displacement Δx is $F_x \Delta x$, which equals the area of the shaded rectangle. The total work done for the displacement from x_i to x_f is approximately equal to the sum of the areas of all the rectangles. (b) The work done by the component F_x of the varying force as the particle moves from x_i to x_f is *exactly* equal to the area under this curve.

Example 7.4 Calculating Total Work Done from a Graph

A force acting on a particle varies with x, as shown in Figure 7.8. Calculate the work done by the force as the particle moves from $x = 0$ to $x = 6.0$ m.

Solution The work done by the force is equal to the area under the curve from $x_A = 0$ to $x_C = 6.0$ m. This area is equal to the area of the rectangular section from Ⓐ to Ⓑ plus the area of the triangular section from Ⓑ to Ⓒ. The area of the rectangle is (5.0 N)(4.0 m) = 20 J, and the area of the triangle is $\frac{1}{2}$(5.0 N)(2.0 m) = 5.0 J. Therefore, the total work done by the force on the particle is $\boxed{25 \text{ J.}}$

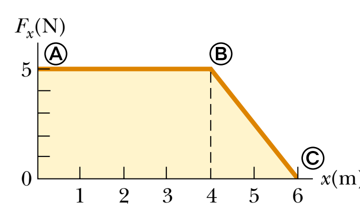


Figure 7.8 (Example 7.4) The force acting on a particle is constant for the first 4.0 m of motion and then decreases linearly with x from $x_B = 4.0$ m to $x_C = 6.0$ m. The net work done by this force is the area under the curve.

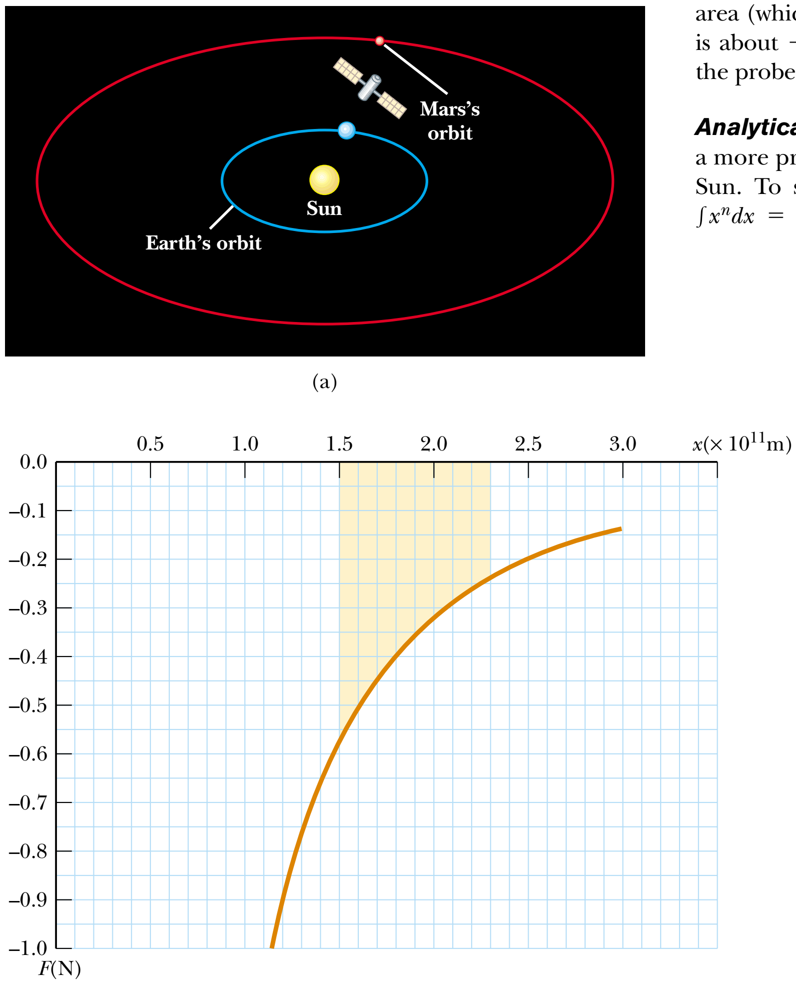
Example 7.5 Work Done by the Sun on a Probe

The interplanetary probe shown in Figure 7.9a is attracted to the Sun by a force given by

$$F = -\frac{1.3 \times 10^{22}}{x^2}$$

in SI units, where x is the Sun-probe separation distance. Graphically and analytically determine how much work is done by the Sun on the probe as the probe–Sun separation changes from 1.5×10^{11} m to 2.3×10^{11} m.

Graphical Solution The negative sign in the equation for the force indicates that the probe is attracted to the Sun. Because the probe is moving away from the Sun, we expect to obtain a negative value for the work done on it. A spreadsheet or other numerical means can be used to generate a graph like that in Figure 7.9b. Each small square of the grid corresponds to an area (0.05 N)(0.1 × 10^{11} m) = 5 × 10^{8} J. The work done is equal to the shaded area in Figure 7.9b. Because there are approximately 60 squares shaded, the total



area (which is negative because the curve is below the x axis) is about -3×10^{10} J. This is the work done by the Sun on the probe.

Analytical Solution We can use Equation 7.7 to calculate a more precise value for the work done on the probe by the Sun. To solve this integral, we make use of the integral $\int x^n dx = x^{n+1}/(n+1)$ with $n = -2$:

$$W = \int_{1.5\times10^{11}}^{2.3\times10^{11}} \left(\frac{-1.3 \times 10^{22}}{x^2}\right) dx$$

$$= (-1.3 \times 10^{22}) \int_{1.5\times10^{11}}^{2.3\times10^{11}} x^{-2} dx$$

$$= (-1.3 \times 10^{22}) \left(\frac{x^{-1}}{-1}\right)\Bigg|_{1.5\times10^{11}}^{2.3\times10^{11}}$$

$$= (-1.3 \times 10^{22}) \left(\frac{-1}{2.3 \times 10^{11}} - \frac{-1}{1.5 \times 10^{11}}\right)$$

$$= \boxed{-3.0 \times 10^{10}\ \text{J}}$$

Figure 7.9 (Example 7.5) (a) An interplanetary probe moves from a position near the Earth's orbit radially outward from the Sun, ending up near the orbit of Mars. (b) Attractive force versus distance for the interplanetary probe.

Work Done by a Spring

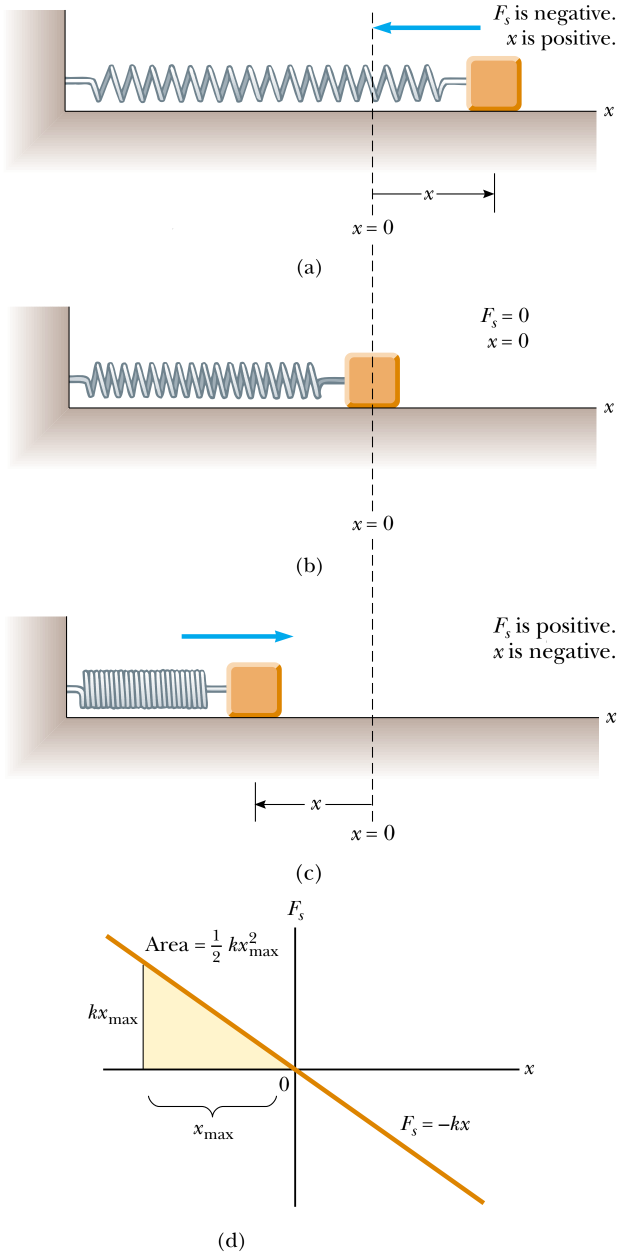
A model of a common physical system for which the force varies with position is shown in Figure 7.10. A block on a horizontal, frictionless surface is connected to a spring. If the spring is either stretched or compressed a small distance from its unstretched (equilibrium) configuration, it exerts on the block a force that can be expressed as

Spring force

$$F_s = -kx \tag{7.9}$$

where x is the position of the block relative to its equilibrium ($x = 0$) position and k is a positive constant called the **force constant** or the **spring constant** of the spring. In other words, the force required to stretch or compress a spring is proportional to the amount of stretch or compression x. This force law for springs is known as **Hooke's law**. The value of k is a measure of the *stiffness* of the spring. Stiff springs have large k values, and soft springs have small k values. As can be seen from Equation 7.9, the units of k are N/m.

The negative sign in Equation 7.9 signifies that the force exerted by the spring is always directed *opposite* to the displacement from equilibrium. When $x > 0$ as in Figure 7.10a, so that the block is to the right of the equilibrium position, the spring force is directed to the left, in the negative x direction. When $x < 0$ as in Figure 7.10c, the block is to the left of equilibrium and the spring force is directed to the right, in the positive x direction. When $x = 0$ as in Figure 7.10b, the spring is unstretched and $F_s = 0$.



Active Figure 7.10 The force exerted by a spring on a block varies with the block's position x relative to the equilibrium position $x = 0$. (a) When x is positive (stretched spring), the spring force is directed to the left. (b) When x is zero (natural length of the spring), the spring force is zero. (c) When x is negative (compressed spring), the spring force is directed to the right. (d) Graph of F_s versus x for the block–spring system. The work done by the spring force as the block moves from $-x_{max}$ to 0 is the area of the shaded triangle, $\frac{1}{2}kx_{max}^2$.

At the Active Figures link at http://www.pse6.com, you can observe the block's motion for various maximum displacements and spring constants.

Because the spring force always acts toward the equilibrium position ($x = 0$), it is sometimes called a *restoring force.* If the spring is compressed until the block is at the point $-x_{max}$ and is then released, the block moves from $-x_{max}$ through zero to $+x_{max}$. If the spring is instead stretched until the block is at the point $+x_{max}$ and is then released, the block moves from $+x_{max}$ through zero to $-x_{max}$. It then reverses direction, returns to $+x_{max}$, and continues oscillating back and forth.

Suppose the block has been pushed to the left to a position $-x_{max}$ and is then released. Let us identify the block as our system and calculate the work W_s done by the spring force on the block as the block moves from $x_i = -x_{max}$ to $x_f = 0$. Applying

Equation 7.7 and assuming the block may be treated as a particle, we obtain

$$W_s = \int_{x_i}^{x_f} F_s dx = \int_{-x_{\max}}^{0} (-kx)\,dx = \tfrac{1}{2}kx_{\max}^2 \tag{7.10}$$

where we have used the integral $\int x^n dx = x^{n+1}/(n+1)$ with $n = 1$. The work done by the spring force is positive because the force is in the same direction as the displacement of the block (both are to the right). Because the block arrives at $x = 0$ with some speed, it will continue moving, until it reaches a position $+x_{\max}$. When we consider the work done by the spring force as the block moves from $x_i = 0$ to $x_f = x_{\max}$, we find that $W_s = -\frac{1}{2}kx_{\max}^2$ because for this part of the motion the displacement is to the right and the spring force is to the left. Therefore, the *net* work done by the spring force as the block moves from $x_i = -x_{\max}$ to $x_f = x_{\max}$ is *zero.*

Figure 7.10d is a plot of F_s versus x. The work calculated in Equation 7.10 is the area of the shaded triangle, corresponding to the displacement from $-x_{\max}$ to 0. Because the triangle has base $x_{\max}$ and height $kx_{\max}$, its area is $\frac{1}{2}kx_{\max}^2$, the work done by the spring as given by Equation 7.10.

If the block undergoes an arbitrary displacement from $x = x_i$ to $x = x_f$, the work done by the spring force on the block is

Work done by a spring

$$W_s = \int_{x_i}^{x_f} (-kx)\,dx = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2 \tag{7.11}$$

For example, if the spring has a force constant of 80 N/m and is compressed 3.0 cm from equilibrium, the work done by the spring force as the block moves from $x_i = -3.0$ cm to its unstretched position $x_f = 0$ is 3.6×10^{-2} J. From Equation 7.11 we also see that the work done by the spring force is zero for any motion that ends where it began ($x_i = x_f$). We shall make use of this important result in Chapter 8, in which we describe the motion of this system in greater detail.

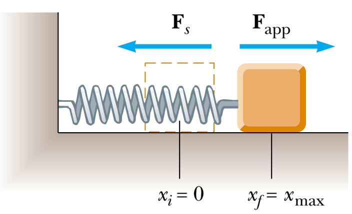


Figure 7.11 A block being pulled from $x_i = 0$ to $x_f = x_{\max}$ on a frictionless surface by a force $\mathbf{F}_{\text{app}}$. If the process is carried out very slowly, the applied force is equal in magnitude and opposite in direction to the spring force at all times.

Equations 7.10 and 7.11 describe the work done by the spring on the block. Now let us consider the work done on the spring by an *external agent* that stretches the spring very slowly from $x_i = 0$ to $x_f = x_{\max}$, as in Figure 7.11. We can calculate this work by noting that at any value of the position, the *applied force* $\mathbf{F}_{\text{app}}$ is equal in magnitude and opposite in direction to the spring force $\mathbf{F}_s$, so that $F_{\text{app}} = -(-kx) = kx$. Therefore, the work done by this applied force (the external agent) on the block–spring system is

$$W_{F_{\text{app}}} = \int_0^{x_{\max}} F_{\text{app}}\,dx = \int_0^{x_{\max}} kx\,dx = \tfrac{1}{2}kx_{\max}^2$$

This work is equal to the negative of the work done by the spring force for this displacement.

The work done by an applied force on a block–spring system between arbitrary positions of the block is

$$W_{F_{\text{app}}} = \int_{x_i}^{x_f} F_{\text{app}}\,dx = \int_{x_i}^{x_f} kx\,dx = \tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2 \tag{7.12}$$

Notice that this is the negative of the work done by the spring as expressed by Equation 7.11. This is consistent with the fact that the spring force and the applied force are of equal magnitude but in opposite directions.

Quick Quiz 7.5 A dart is loaded into a spring-loaded toy dart gun by pushing the spring in by a distance d. For the next loading, the spring is compressed a distance $2d$. How much work is required to load the second dart compared to that required to load the first? (a) four times as much (b) two times as much (c) the same (d) half as much (e) one-fourth as much.

Example 7.6 Measuring *k* for a Spring

A common technique used to measure the force constant of a spring is demonstrated by the setup in Figure 7.12. The spring is hung vertically, and an object of mass m is attached to its lower end. Under the action of the "load" mg, the spring stretches a distance d from its equilibrium position.

(A) If a spring is stretched 2.0 cm by a suspended object having a mass of 0.55 kg, what is the force constant of the spring?

Solution Because the object (the system) is at rest, the upward spring force balances the downward gravitational force $m\mathbf{g}$. In this case, we apply Hooke's law to give $|\mathbf{F}_s| = kd = mg$, or

$$k = \frac{mg}{d} = \frac{(0.55\ \text{kg})(9.80\ \text{m/s}^2)}{2.0 \times 10^{-2}\ \text{m}} = \boxed{2.7 \times 10^2\ \text{N/m}}$$

(B) How much work is done by the spring as it stretches through this distance?

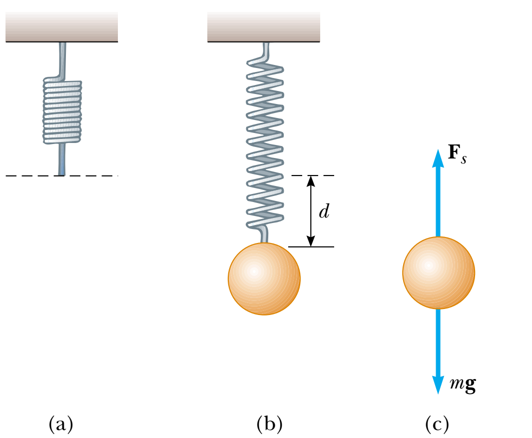


Figure 7.12 (Example 7.6) Determining the force constant k of a spring. The elongation d is caused by the attached object, which has a weight mg. Because the spring force balances the gravitational force, it follows that $k = mg/d$.

Solution Using Equation 7.11,

$$W_s = 0 - \tfrac{1}{2}kd^2 = -\tfrac{1}{2}(2.7 \times 10^2\ \text{N/m})(2.0 \times 10^{-2}\ \text{m})^2$$
$$= \boxed{-5.4 \times 10^{-2}\ \text{J}}$$

What If? Suppose this measurement is made on an elevator with an upward vertical acceleration a. Will the unaware experimenter arrive at the same value of the spring constant?

Answer The force $\mathbf{F}_s$ in Figure 7.12 must be larger than $m\mathbf{g}$ to produce an upward acceleration of the object. Because $\mathbf{F}_s$ must increase in magnitude, and $|\mathbf{F}_s| = kd$, the spring must extend farther. The experimenter sees a larger extension for the same hanging weight and therefore measures the spring constant to be smaller than the value found in part (A) for $\mathbf{a} = 0$.

Newton's second law applied to the hanging object gives

$$\sum F_y = |\mathbf{F}_s| - mg = ma_y$$
$$kd - mg = ma_y$$
$$d = \frac{m(g + a_y)}{k}$$

where k is the *actual* spring constant. Now, the experimenter is unaware of the acceleration, so she claims that $|\mathbf{F}_s| = k'd = mg$ where k' is the spring constant as measured by the experimenter. Thus,

$$k' = \frac{mg}{d} = \frac{mg}{\left(\dfrac{m(g + a_y)}{k}\right)} = \frac{g}{g + a_y}k$$

If the acceleration of the elevator is upward so that a_y is positive, this result shows that the measured spring constant will be smaller, consistent with our conceptual argument.

7.5 Kinetic Energy and the Work–Kinetic Energy Theorem

We have investigated work and identified it as a mechanism for transferring energy into a system. One of the possible outcomes of doing work on a system is that the system changes its speed. In this section, we investigate this situation and introduce our first type of energy that a system can possess, called *kinetic energy*.

Consider a system consisting of a single object. Figure 7.13 shows a block of mass m moving through a displacement directed to the right under the action of a net force $\Sigma\mathbf{F}$, also directed to the right. We know from Newton's second law that the block moves with an acceleration $\mathbf{a}$. If the block moves through a displacement $\Delta\mathbf{r} = \Delta x\hat{\mathbf{i}} = (x_f - x_i)\hat{\mathbf{i}}$, the work done by the net force $\Sigma\mathbf{F}$ is

$$\sum W = \int_{x_i}^{x_f} \sum F\,dx \qquad (7.13)$$

Using Newton's second law, we can substitute for the magnitude of the net force $\Sigma F = ma$, and then perform the following chain-rule manipulations on the integrand:

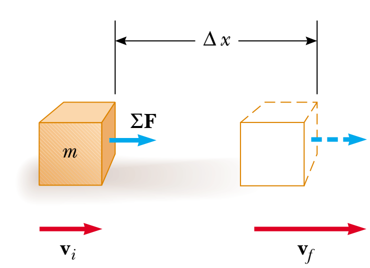


Figure 7.13 An object undergoing a displacement $\Delta\mathbf{r} = \Delta x\hat{\mathbf{i}}$ and a change in velocity under the action of a constant net force $\Sigma\mathbf{F}$.

$$\sum W = \int_{x_i}^{x_f} ma\,dx = \int_{x_i}^{x_f} m\,\frac{dv}{dt}\,dx = \int_{x_i}^{x_f} m\,\frac{dv}{dx}\,\frac{dx}{dt}\,dx = \int_{v_i}^{v_f} mv\,dv$$

$$\sum W = \tfrac{1}{2}mv_f^{\,2} - \tfrac{1}{2}mv_i^{\,2} \tag{7.14}$$

where v_i is the speed of the block when it is at $x = x_i$ and v_f is its speed at x_f.

This equation was generated for the specific situation of one-dimensional motion, but it is a general result. It tells us that the work done by the net force on a particle of mass m is equal to the difference between the initial and final values of a quantity $\frac{1}{2}mv^2$. The quantity $\frac{1}{2}mv^2$ represents the energy associated with the motion of the particle. This quantity is so important that it has been given a special name—**kinetic energy.** Equation 7.14 states that the net work done on a particle by a net force $\Sigma\mathbf{F}$ acting on it equals the change in kinetic energy of the particle.

In general, the kinetic energy K of a particle of mass m moving with a speed v is defined as

Kinetic energy

$$K \equiv \tfrac{1}{2}mv^2 \tag{7.15}$$

Kinetic energy is a scalar quantity and has the same units as work. For example, a 2.0 kg object moving with a speed of 4.0 m/s has a kinetic energy of 16 J. Table 7.1 lists the kinetic energies for various objects.

It is often convenient to write Equation 7.14 in the form

Work–kinetic energy theorem

$$\sum W = K_f - K_i = \Delta K \tag{7.16}$$

Another way to write this is $K_f = K_i + \Sigma W$, which tells us that the final kinetic energy is equal to the initial kinetic energy plus the change due to the work done.

Equation 7.16 is an important result known as the **work–kinetic energy theorem:**

In the case in which work is done on a system and the only change in the system is in its speed, the work done by the net force equals the change in kinetic energy of the system.

The work–kinetic energy theorem indicates that the speed of a particle will *increase* if the net work done on it is *positive,* because the final kinetic energy will be greater than the initial kinetic energy. The speed will *decrease* if the net work is *negative,* because the final kinetic energy will be less than the initial kinetic energy.

PITFALL PREVENTION

7.6 Conditions for the Work–Kinetic Energy Theorem

The work–kinetic energy theorem is important, but limited in its application—it is not a general principle. There are many situations in which other changes in the system occur besides its speed, and there are other interactions with the environment besides work. A more general principle involving energy is conservation of energy in Section 7.6.

Table 7.1

Kinetic Energies for Various Objects

Object	Mass (kg)	Speed (m/s)	Kinetic Energy (J)
Earth orbiting the Sun	5.98×10^{24}	2.98×10^{4}	2.66×10^{33}
Moon orbiting the Earth	7.35×10^{22}	1.02×10^{3}	3.82×10^{28}
Rocket moving at escape speed[a]	500	1.12×10^{4}	3.14×10^{10}
Automobile at 65 mi/h	2 000	29	8.4×10^{5}
Running athlete	70	10	3 500
Stone dropped from 10 m	1.0	14	98
Golf ball at terminal speed	0.046	44	45
Raindrop at terminal speed	3.5×10^{-5}	9.0	1.4×10^{-3}
Oxygen molecule in air	5.3×10^{-26}	500	6.6×10^{-21}

[a] Escape speed is the minimum speed an object must reach near the Earth's surface in order to move infinitely far away from the Earth.

Because we have only investigated translational motion through space so far, we arrived at the work–kinetic energy theorem by analyzing situations involving translational motion. Another type of motion is *rotational motion,* in which an object spins about an axis. We will study this type of motion in Chapter 10. The work–kinetic energy theorem is also valid for systems that undergo a change in the rotational speed due to work done on the system. The windmill in the chapter opening photograph is an example of work causing rotational motion.

The work–kinetic energy theorem will clarify a result that we have seen earlier in this chapter that may have seemed odd. In Section 7.4, we arrived at a result of zero net work done when we let a spring push a block from $x_i = -x_{max}$ to $x_f = x_{max}$. Notice that the speed of the block is continually changing during this process, so it may seem complicated to analyze this process. The quantity ΔK in the work–kinetic energy theorem, however, only refers to the initial and final points for the speeds—it does not depend on details of the path followed between these points. Thus, because the speed is zero at both the initial and final points of the motion, the net work done on the block is zero. We will see this concept of path independence often in similar approaches to problems.

Earlier, we indicated that work can be considered as a mechanism for transferring energy into a system. Equation 7.16 is a mathematical statement of this concept. We do work $\sum W$ on a system and the result is a transfer of energy across the boundary of the system. The result on the system, in the case of Equation 7.16, is a change ΔK in kinetic energy. We will explore this idea more fully in the next section.

PITFALL PREVENTION

7.7 The Work–Kinetic Energy Theorem: Speed, not Velocity

The work–kinetic energy theorem relates work to a change in the *speed* of an object, not a change in its velocity. For example, if an object is in uniform circular motion, the speed is constant. Even though the velocity is changing, no work is done by the force causing the circular motion.

Quick Quiz 7.6 A dart is loaded into a spring-loaded toy dart gun by pushing the spring in by a distance *d*. For the next loading, the spring is compressed a distance 2*d*. How much faster does the second dart leave the gun compared to the first? (a) four times as fast (b) two times as fast (c) the same (d) half as fast (e) one-fourth as fast.

Example 7.7 A Block Pulled on a Frictionless Surface

A 6.0-kg block initially at rest is pulled to the right along a horizontal, frictionless surface by a constant horizontal force of 12 N. Find the speed of the block after it has moved 3.0 m.

Solution We have made a drawing of this situation in Figure 7.14. We could apply the equations of kinematics to determine the answer, but let us practice the energy approach. The block is the system, and there are three external forces acting on the system. The normal force balances the gravitational force on the block, and neither of these vertically acting forces does work on the block because their points of application are horizontally displaced. Thus, the net external force acting on the block is the 12-N force. The work done by this force is

$$W = F\Delta x = (12\,\text{N})(3.0\,\text{m}) = 36\,\text{J}$$

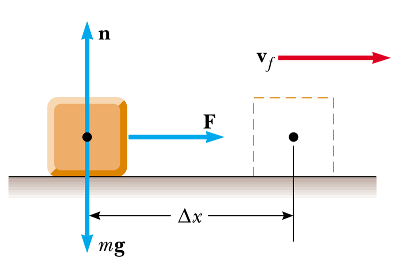


Figure 7.14 (Example 7.7) A block pulled to the right on a frictionless surface by a constant horizontal force.

Using the work–kinetic energy theorem and noting that the initial kinetic energy is zero, we obtain

$$W = K_f - K_i = \tfrac{1}{2}mv_f^2 - 0$$

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(36\,\text{J})}{6.0\,\text{kg}}} = \boxed{3.5\,\text{m/s}}$$

What If? **Suppose the magnitude of the force in this example is doubled to $F' = 2F$. The 6.0-kg block accelerates to 3.5 m/s due to this applied force while moving through a displacement $\Delta x'$. (A) How does the displacement $\Delta x'$ compare to the original displacement Δx? (B) How does the time interval $\Delta t'$ for the block to accelerate from rest to 3.5 m/s compare to the original interval Δt?**

Answer (A) If we pull harder, the block should accelerate to a higher speed in a shorter distance, so we expect $\Delta x' < \Delta x$. Mathematically, from the work–kinetic energy theorem $W = \Delta K$, we find

$$F'\Delta x' = \Delta K = F\Delta x$$

$$\Delta x' = \frac{F}{F'}\Delta x = \frac{F}{2F}\Delta x = \tfrac{1}{2}\Delta x$$

and the distance is shorter as suggested by our conceptual argument.

(B) If we pull harder, the block should accelerate to a higher speed in a shorter time interval, so we expect $\Delta t' < \Delta t$. Mathematically, from the definition of average velocity,

$$\bar{v} = \frac{\Delta x}{\Delta t} \quad \longrightarrow \quad \Delta t = \frac{\Delta x}{\bar{v}}$$

Because both the original force and the doubled force cause the same change in velocity, the average velocity $\bar{v}$ is the same in both cases. Thus,

$$\Delta t' = \frac{\Delta x'}{\bar{v}} = \frac{\frac{1}{2}\Delta x}{\bar{v}} = \frac{1}{2}\Delta t$$

and the time interval is shorter, consistent with our conceptual argument.

Conceptual Example 7.8 Does the Ramp Lessen the Work Required?

A man wishes to load a refrigerator onto a truck using a ramp, as shown in Figure 7.15. He claims that less work would be required to load the truck if the length L of the ramp were increased. Is his statement valid?

Solution No. Suppose the refrigerator is wheeled on a dolly up the ramp at constant speed. Thus, $\Delta K = 0$. The normal force exerted by the ramp on the refrigerator is directed at 90° to the displacement and so does no work on the refrigerator. Because $\Delta K = 0$, the work–kinetic energy theorem gives

$$W_{\text{net}} = W_{\text{by man}} + W_{\text{by gravity}} = 0$$

The work done by the gravitational force equals the product of the weight mg of the refrigerator, the height h through which it is displaced, and cos 180°, or $W_{\text{by gravity}} = -mgh$. (The negative sign arises because the downward gravitational force is opposite the displacement.) Thus, the man must do the same amount of work mgh on the refrigerator, *regardless* of the length of the ramp. Although less force is required with a longer ramp, that force must act over a greater distance.

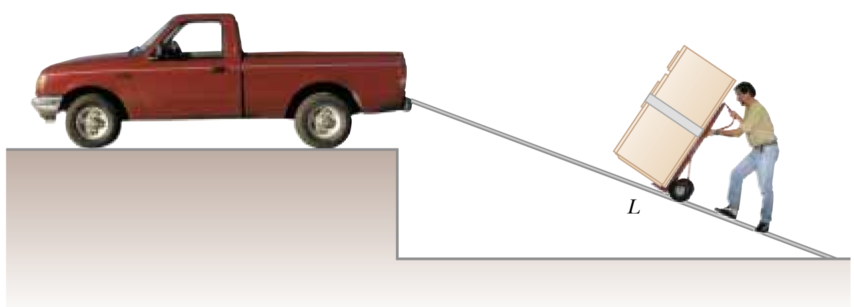


Figure 7.15 (Conceptual Example 7.8) A refrigerator attached to a frictionless wheeled dolly is moved up a ramp at constant speed.

7.6 The Nonisolated System—Conservation of Energy

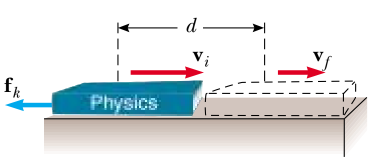


Figure 7.16 A book sliding to the right on a horizontal surface slows down in the presence of a force of kinetic friction acting to the left. The initial velocity of the book is $\mathbf{v}_i$, and its final velocity is $\mathbf{v}_f$. The normal force and the gravitational force are not included in the diagram because they are perpendicular to the direction of motion and therefore do not influence the book's speed.

We have seen examples in which an object, modeled as a particle, is acted on by various forces, resulting in a change in its kinetic energy. This very simple situation is the first example of the **nonisolated system**—a common scenario in physics problems. Physical problems for which this scenario is appropriate involve systems that interact with or are influenced by their environment, causing some kind of change in the system. If a system does not interact with its environment it is an **isolated system**, which we will study in Chapter 8.

The work–kinetic energy theorem is our first example of an energy equation appropriate for a nonisolated system. In the case of the work–kinetic energy theorem, the interaction is the work done by the external force, and the quantity in the system that changes is the kinetic energy.

In addition to kinetic energy, we now introduce a second type of energy that a system can possess. Let us imagine the book in Figure 7.16 sliding to the right on the surface of a heavy table and slowing down due to the friction force. Suppose the *surface* is the system. Then the friction force from the sliding book does work on the surface. The force on the surface is to the right and the displacement of the point of application of the force is to the right—the work is positive. But the surface is not moving after the book has stopped. Positive work has been done on the surface, yet there is no increase in the surface's kinetic energy. Is this a violation of the work–kinetic energy theorem?

It is not really a violation, because this situation does not fit the description of the conditions given for the work–kinetic energy theorem. Work is done on the system of the surface, but the result of that work is *not* an increase in kinetic energy. From your everyday experience with sliding over surfaces with friction, you can probably guess that the surface will be *warmer* after the book slides over it. (Rub your hands together briskly to experience this!) Thus, the work that was done on the surface has gone into warming the surface rather than increasing its speed. We call the energy associated with an object's temperature its **internal energy,** symbolized E_{int}. (We will define internal energy more generally in Chapter 20.) In this case, the work done on the surface does indeed represent energy transferred into the system, but it appears in the system as internal energy rather than kinetic energy.

We have now seen two methods of storing energy in a system—kinetic energy, related to motion of the system, and internal energy, related to its temperature. A third method, which we cover in Chapter 8, is *potential energy*. This is energy related to the configuration of a system in which the components of the system interact by forces. For example, when a spring is stretched, *elastic potential energy* is stored in the spring due to the force of interaction between the spring coils. Other types of potential energy include gravitational and electric.

We have seen only one way to transfer energy into a system so far—work. We mention below a few other ways to transfer energy into or out of a system. The details of these processes will be studied in other sections of the book. We illustrate these in Figure 7.17 and summarize them as follows:

Work, as we have learned in this chapter, is a method of transferring energy to a system by applying a force to the system and causing a displacement of the point of application of the force (Fig. 7.17a).

Mechanical waves (Chapters 16–18) are a means of transferring energy by allowing a disturbance to propagate through air or another medium. This is the method by which energy (which you detect as sound) leaves your clock radio through the loudspeaker and enters your ears to stimulate the hearing process (Fig. 7.17b). Other examples of mechanical waves are seismic waves and ocean waves.

Heat (Chapter 20) is a mechanism of energy transfer that is driven by a temperature difference between two regions in space. One clear example is thermal conduction, a mechanism of transferring energy by microscopic collisions. For example, a metal spoon in a cup of coffee becomes hot because fast-moving electrons and atoms in the submerged portion of the spoon bump into slower ones in the nearby part of the handle (Fig. 7.17c). These particles move faster because of the collisions and bump into the next group of slow particles. Thus, the internal energy of the spoon handle rises from energy transfer due to this bumping process.[4]

Matter transfer (Chapter 20) involves situations in which matter physically crosses the boundary of a system, carrying energy with it. Examples include filling your automobile tank with gasoline (Fig. 7.17d), and carrying energy to the rooms of your home by circulating warm air from the furnace, a process called *convection*.

[4] The process we call heat can also proceed by convection and radiation, as well as conduction. Convection and radiation, described in Chapter 20, overlap with other types of energy transfer in our list of six.

PITFALL PREVENTION

7.8 Heat is not a Form of Energy

The word *heat* is one of the most misused words in our popular language. In this text, heat is a method of *transferring* energy, *not* a form of storing energy. Thus, phrases such as "heat content," "the heat of the summer," and "the heat escaped" all represent uses of this word that are inconsistent with our physics definition. See Chapter 20.

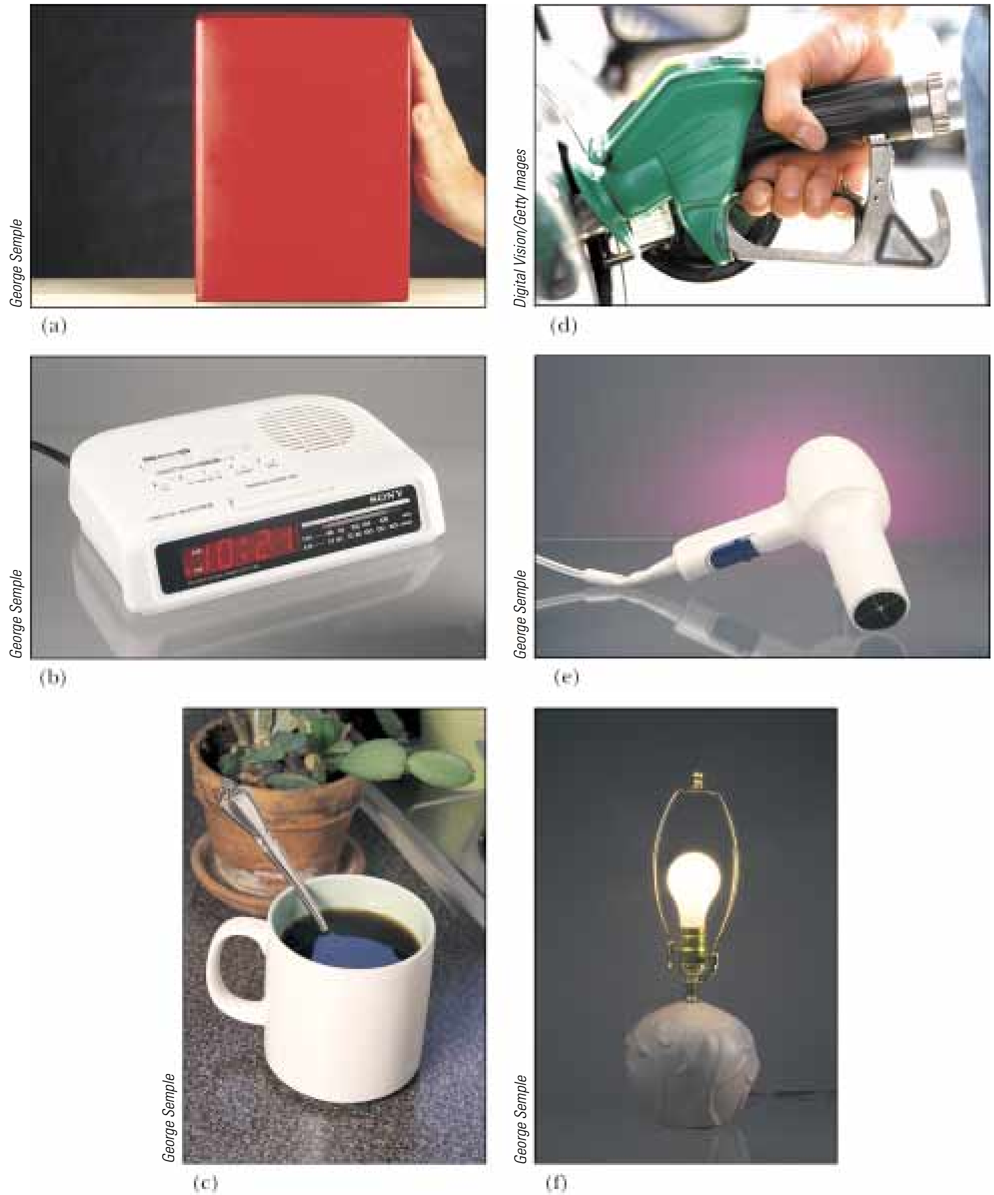


Figure 7.17 Energy transfer mechanisms. (a) Energy is transferred to the block by *work*; (b) energy leaves the radio from the speaker by *mechanical waves*; (c) energy transfers up the handle of the spoon by *heat*; (d) energy enters the automobile gas tank by *matter transfer*; (e) energy enters the hair dryer by *electrical transmission*; and (f) energy leaves the light bulb by *electromagnetic radiation*.

Electrical Transmission (Chapters 27–28) involves energy transfer by means of electric currents. This is how energy transfers into your hair dryer (Fig. 7.17e), stereo system, or any other electrical device.

Electromagnetic radiation (Chapter 34) refers to electromagnetic waves such as light, microwaves, radio waves, and so on (Fig. 7.17f). Examples of this method of transfer include cooking a baked potato in your microwave oven and light energy traveling from the Sun to the Earth through space.[5]

[5] Electromagnetic radiation and work done by field forces are the only energy transfer mechanisms that do not require molecules of the environment to be available at the system boundary. Thus, systems surrounded by a vacuum (such as planets) can only exchange energy with the environment by means of these two possibilities.

One of the central features of the energy approach is the notion that **we can neither create nor destroy energy—energy is always *conserved*.** Thus, **if the total amount of energy in a system changes, it can *only* be due to the fact that energy has crossed the boundary of the system by a transfer mechanism such as one of the methods listed above.** This is a general statement of the principle of **conservation of energy.** We can describe this idea mathematically as follows:

$$\Delta E_{\text{system}} = \sum T \tag{7.17}$$

Conservation of energy

where E_{system} is the total energy of the system, including all methods of energy storage (kinetic, internal, and potential, as discussed in Chapter 8) and T is the amount of energy transferred across the system boundary by some mechanism. Two of our transfer mechanisms have well-established symbolic notations. For work, $T_{\text{work}} = W$, as we have seen in the current chapter, and for heat, $T_{\text{heat}} = Q$, as defined in Chapter 20. The other four members of our list do not have established symbols.

This is no more complicated in theory than is balancing your checking account statement. If your account is the system, the change in the account balance for a given month is the sum of all the transfers—deposits, withdrawals, fees, interest, and checks written. It may be useful for you to think of energy as the *currency of nature*!

Suppose a force is applied to a nonisolated system and the point of application of the force moves through a displacement. Suppose further that the only effect on the system is to change its speed. Then the only transfer mechanism is work (so that ΣT in Equation 7.17 reduces to just W) and the only kind of energy in the system that changes is the kinetic energy (so that ΔE_{system} reduces to just ΔK). Equation 7.17 then becomes

$$\Delta K = W$$

which is the work–kinetic energy theorem. The work–kinetic energy theorem is a special case of the more general principle of conservation of energy. We shall see several more special cases in future chapters.

Quick Quiz 7.7 By what transfer mechanisms does energy enter and leave (a) your television set; (b) your gasoline-powered lawn mower; (c) your hand-cranked pencil sharpener?

Quick Quiz 7.8 Consider a block sliding over a horizontal surface with friction. Ignore any sound the sliding might make. If we consider the system to be the *block*, this system is (a) isolated (b) nonisolated (c) impossible to determine.

Quick Quiz 7.9 If we consider the system in Quick Quiz 7.8 to be the *surface*, this system is (a) isolated (b) nonisolated (c) impossible to determine.

Quick Quiz 7.10 If we consider the system in Quick Quiz 7.8 to be the *block and the surface*, this system is (a) isolated (b) nonisolated (c) impossible to determine.

7.7 Situations Involving Kinetic Friction

Consider again the book in Figure 7.16 sliding to the right on the surface of a heavy table and slowing down due to the friction force. Work is done by the friction force because there is a force and a displacement. Keep in mind, however, that our equations for work involve the displacement *of the point of application of the force*. The friction force is spread out over the entire contact area of an object sliding on a surface, so the force is not localized at a point. In addition, the magnitudes of the friction forces at various points are constantly changing as spot welds occur, the surface and the book deform locally, and so on. The points of application of the friction force on the book are jumping all over the face of the book in contact with the surface. This means that the displacement of the point of application of the friction force (assuming we could calculate it!) is not the same as the displacement of the book.

The work–kinetic energy theorem is valid for a particle or an object that can be modeled as a particle. When an object cannot be treated as a particle, however, things become more complicated. For these kinds of situations, Newton's second law is still valid for the system, even though the work–kinetic energy theorem is not. In the case of a nondeformable object like our book sliding on the surface,[6] we can handle this in a relatively straightforward way.

Starting from a situation in which a constant force is applied to the book, we can follow a similar procedure to that in developing Equation 7.14. We start by multiplying each side of Newton's second law (x component only) by a displacement Δx of the book:

$$\left(\sum F_x\right)\Delta x = (ma_x)\Delta x \tag{7.18}$$

For a particle under constant acceleration, we know that the following relationships (Eqs. 2.9 and 2.11) are valid:

$$a_x = \frac{v_f - v_i}{t} \qquad \Delta x = \tfrac{1}{2}(v_i + v_f)\,t$$

where v_i is the speed at $t = 0$ and v_f is the speed at time t. Substituting these expressions into Equation 7.18 gives

$$\left(\sum F_x\right)\Delta x = m\left(\frac{v_f - v_i}{t}\right)\tfrac{1}{2}(v_i + v_f)\,t$$

$$\left(\sum F_x\right)\Delta x = \tfrac{1}{2}mv_f^{\,2} - \tfrac{1}{2}mv_i^{\,2}$$

This *looks* like the work–kinetic energy theorem, but *the left hand side has not been called work*. The quantity Δx is the displacement of the book—*not* the displacement of the point of application of the friction force.

Let us now apply this equation to a book that has been projected across a surface. We imagine that the book has an initial speed and slows down due to friction, the only force in the horizontal direction. The net force on the book is the kinetic friction force $\mathbf{f}_k$, which is directed opposite to the displacement Δx. Thus,

$$\left(\sum F_x\right)\Delta x = -f_k\,\Delta x = \tfrac{1}{2}mv_f^{\,2} - \tfrac{1}{2}mv_i^{\,2} = \Delta K$$

$$-f_k\,\Delta x = \Delta K \tag{7.19}$$

which mathematically describes the decrease in kinetic energy due to the friction force.

We have generated these results by assuming that a book is moving along a straight line. An object could also slide over a surface with friction and follow a curved path. In this case, Equation 7.19 must be generalized as follows:

Change in kinetic energy due to friction

$$-f_k\,d = \Delta K \tag{7.20}$$

where d is the length of the path followed by an object.

If there are other forces besides friction acting on an object, the change in kinetic energy is the sum of that due to the other forces from the work–kinetic energy theorem, and that due to friction:

[6] The overall shape of the book remains the same, which is why we are saying it is nondeformable. On a microscopic level, however, there is deformation of the book's face as it slides over the surface.

$$\Delta K = -f_k d + \sum W_{\text{other forces}} \quad (7.21a)$$

or

$$K_f = K_i - f_k d + \sum W_{\text{other forces}} \quad (7.21b)$$

Now consider the larger system of the book *and* the surface as the book slows down under the influence of a friction force alone. There is no work done across the boundary of this system—the system does not interact with the environment. There are no other types of energy transfer occurring across the boundary of the system, assuming we ignore the inevitable sound the sliding book makes! In this case, Equation 7.17 becomes

$$\Delta E_{\text{system}} = \Delta K + \Delta E_{\text{int}} = 0$$

The change in kinetic energy of this book-plus-surface system is the same as the change in kinetic energy of the the book alone in Equation 7.20, because the book is the only part of the book-surface system that is moving. Thus,

$$-f_k d + \Delta E_{\text{int}} = 0$$

$$\Delta E_{\text{int}} = f_k d \quad (7.22)$$

Change in internal energy due to friction

Thus, the increase in internal energy of the system is equal to the product of the friction force and the displacement of the book.

The conclusion of this discussion is that **the result of a friction force is to transform kinetic energy into internal energy, and the increase in internal energy is equal to the decrease in kinetic energy.**

Quick Quiz 7.11 You are traveling along a freeway at 65 mi/h. Your car has kinetic energy. You suddenly skid to a stop because of congestion in traffic. Where is the kinetic energy that your car once had? (a) All of it is in internal energy in the road. (b) All of it is in internal energy in the tires. (c) Some of it has transformed to internal energy and some of it transferred away by mechanical waves. (d) All of it is transferred away from your car by various mechanisms.

Example 7.9 A Block Pulled on a Rough Surface — Interactive

A 6.0-kg block initially at rest is pulled to the right along a horizontal surface by a constant horizontal force of 12 N.

(A) Find the speed of the block after it has moved 3.0 m if the surfaces in contact have a coefficient of kinetic friction of 0.15. (This is Example 7.7, modified so that the surface is no longer frictionless.)

Solution Conceptualize this problem by realizing that the rough surface is going to apply a friction force opposite to the applied force. As a result, we expect the speed to be lower than that found in Example 7.7. The surface is rough and we are given forces and a distance, so we categorize this as a situation involving kinetic friction that must be handled by means of Equation 7.21. To analyze the problem, we have made a drawing of this situation in Figure 7.18a. We identify the block as the system, and there are four external forces interacting with the system. The normal force balances the gravitational force on the block, and neither of these vertically acting forces does work on the block because their points of application are displaced horizontally. The applied force does work just as in Example 7.7:

$$W = F\Delta x = (12\ \text{N})(3.0\ \text{m}) = 36\ \text{J}$$

In this case we must use Equation 7.21a to calculate the kinetic energy change due to friction, $\Delta K_{\text{friction}}$. Because the block is in equilibrium in the vertical direction, the normal force $\mathbf{n}$ counterbalances the gravitational force $m\mathbf{g}$, so we have $n = mg$. Hence, the magnitude of the friction force is

$$f_k = \mu_k n = \mu_k mg = (0.15)(6.0\ \text{kg})(9.80\ \text{m/s}^2) = 8.82\ \text{N}$$

The change in kinetic energy of the block due to friction is

$$\Delta K_{\text{friction}} = -f_k d = -(8.82\,\text{N})(3.0\,\text{m}) = -26.5\,\text{J}$$

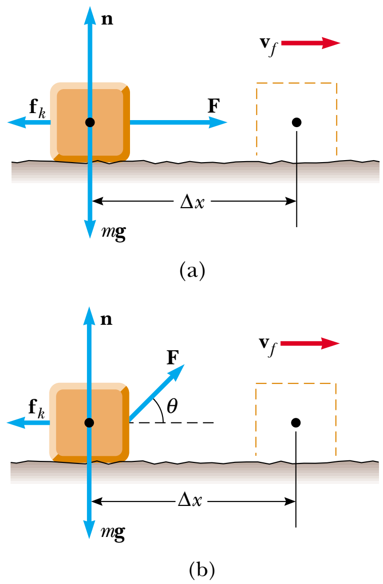


Figure 7.18 (Example 7.9) (a) A block pulled to the right on a rough surface by a constant horizontal force. (b) The applied force is at an angle θ to the horizontal.

The final speed of the block follows from Equation 7.21b:

$$\tfrac{1}{2}mv_f^{\,2} = \tfrac{1}{2}mv_i^{\,2} - f_k d + \sum W_{\text{other forces}}$$

$$v_f = \sqrt{v_i^{\,2} + \frac{2}{m}\left(-f_k d + \sum W_{\text{other forces}}\right)}$$

$$= \sqrt{0 + \frac{2}{6.0\,\text{kg}}(-26.5\,\text{J} + 36\,\text{J})}$$

$$= \boxed{1.8\,\text{m/s}}$$

To finalize this problem note that, after covering the same distance on a frictionless surface (see Example 7.7), the speed of the block was 3.5 m/s.

(B) Suppose the force $\mathbf{F}$ is applied at an angle θ as shown in Figure 7.18b. At what angle should the force be applied to achieve the largest possible speed after the block has moved 3.0 m to the right?

Solution The work done by the applied force is now

$$W = F\Delta x \cos\theta = Fd\cos\theta$$

where $\Delta x = d$ because the path followed by the block is a straight line. The block is in equilibrium in the vertical direction, so

$$\sum F_y = n + F\sin\theta - mg = 0$$

and

$$n = mg - F\sin\theta$$

Because $K_i = 0$, Equation 7.21b can be written,

$$\begin{aligned} K_f &= -f_k d + \sum W_{\text{other forces}} \\ &= -\mu_k n d + Fd\cos\theta \\ &= -\mu_k(mg - F\sin\theta)\,d + Fd\cos\theta \end{aligned}$$

Maximizing the speed is equivalent to maximizing the final kinetic energy. Consequently, we differentiate K_f with respect to θ and set the result equal to zero:

$$\frac{d(K_f)}{d\theta} = -\mu_k(0 - F\cos\theta)\,d - Fd\sin\theta = 0$$

$$\mu_k\cos\theta - \sin\theta = 0$$

$$\tan\theta = \mu_k$$

For $\mu_k = 0.15$, we have,

$$\theta = \tan^{-1}(\mu_k) = \tan^{-1}(0.15) = \boxed{8.5^\circ}$$

Try out the effects of pulling the block at various angles at the Interactive Worked Example link at **http://www.pse6.com.**

Conceptual Example 7.10 Useful Physics for Safer Driving

A car traveling at an initial speed v slides a distance d to a halt after its brakes lock. Assuming that the car's initial speed is instead $2v$ at the moment the brakes lock, estimate the distance it slides.

Solution Let us assume that the force of kinetic friction between the car and the road surface is constant and the same for both speeds. According to Equation 7.20, the friction force multiplied by the distance d is equal to the initial kinetic energy of the car (because $K_f = 0$). If the speed is doubled, as it is in this example, the kinetic energy is quadrupled. For a given friction force, the distance traveled is four times as great when the initial speed is doubled, and so the estimated distance that the car slides is $4d$.

Example 7.11 A Block–Spring System

Interactive

A block of mass 1.6 kg is attached to a horizontal spring that has a force constant of 1.0×10^3 N/m, as shown in Figure 7.10. The spring is compressed 2.0 cm and is then released from rest.

(A) Calculate the speed of the block as it passes through the equilibrium position $x = 0$ if the surface is frictionless.

Solution In this situation, the block starts with $v_i = 0$ at $x_i = -2.0$ cm, and we want to find v_f at $x_f = 0$. We use Equation 7.10 to find the work done by the spring with $x_{\text{max}} = x_i = -2.0\text{ cm} = -2.0 \times 10^{-2}$ m:

$$W_s = \tfrac{1}{2}kx_{\text{max}}^2 = \tfrac{1}{2}(1.0 \times 10^3\,\text{N/m})(-2.0 \times 10^{-2}\,\text{m})^2 = 0.20\,\text{J}$$

Using the work–kinetic energy theorem with $v_i = 0$, we set the change in kinetic energy of the block equal to the work done on it by the spring:

$$W_s = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

$$v_f = \sqrt{v_i^2 + \frac{2}{m}W_s}$$

$$= \sqrt{0 + \frac{2}{1.6 \text{ kg}}(0.20 \text{ J})}$$

$$= \boxed{0.50 \text{ m/s}}$$

(B) Calculate the speed of the block as it passes through the equilibrium position if a constant friction force of 4.0 N retards its motion from the moment it is released.

Solution Certainly, the answer has to be less than what we found in part (A) because the friction force retards the motion. We use Equation 7.20 to calculate the kinetic energy lost because of friction and add this negative value to the kinetic energy we calculated in the absence of friction. The kinetic energy lost due to friction is

$$\Delta K = -f_k d = -(4.0 \text{ N})(2.0 \times 10^{-2} \text{ m}) = -0.080 \text{ J}$$

In part (A), the work done by the spring was found to be 0.20 J. Therefore, the final kinetic energy in the presence of friction is

$$K_f = 0.20 \text{ J} - 0.080 \text{ J} = 0.12 \text{ J} = \tfrac{1}{2}mv_f^2$$

$$v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(0.12 \text{ J})}{1.6 \text{ kg}}} = \boxed{0.39 \text{ m/s}}$$

As expected, this value is somewhat less than the 0.50 m/s we found in part (A). If the friction force were greater, then the value we obtained as our answer would have been even smaller.

What If? **What if the friction force were increased to 10.0 N? What is the block's speed at $x = 0$?**

Answer In this case, the loss of kinetic energy as the block moves to $x = 0$ is

$$\Delta K = -f_k d = -(10.0 \text{ N})(2.0 \times 10^{-2} \text{ m}) = -0.20 \text{ J}$$

which is equal in magnitude to the kinetic energy at $x = 0$ without the loss due to friction. Thus, all of the kinetic energy has been transformed by friction when the block arrives at $x = 0$ and its speed at this point is $v = 0$.

In this situation as well as that in part (B), the speed of the block reaches a maximum at some position other than $x = 0$. Problem 70 asks you to locate these positions.

Investigate the role of the spring constant, amount of spring compression, and surface friction at the Interactive Worked Example link at **http://www.pse6.com.**

7.8 Power

Consider Conceptual Example 7.8 again, which involved rolling a refrigerator up a ramp into a truck. Suppose that the man is not convinced by our argument that the work is the same regardless of the length of the ramp and sets up a long ramp with a gentle rise. Although he will do the same amount of work as someone using a shorter ramp, he will take longer to do the work simply because he has to move the refrigerator over a greater distance. While the work done on both ramps is the same, there is *something* different about the tasks—the *time interval* during which the work is done.

The time rate of energy transfer is called **power.** We will focus on work as the energy transfer method in this discussion, but keep in mind that the notion of power is valid for *any* means of energy transfer. If an external force is applied to an object (which we assume acts as a particle), and if the work done by this force in the time interval Δt is W, then the **average power** during this interval is defined as

$$\overline{\mathcal{P}} \equiv \frac{W}{\Delta t}$$

Thus, while the same work is done in rolling the refrigerator up both ramps, less power is required for the longer ramp.

In a manner similar to the way we approached the definition of velocity and acceleration, we define the **instantaneous power** $\mathcal{P}$ as the limiting value of the average power as Δt approaches zero:

$$\mathcal{P} \equiv \lim_{\Delta t \to 0} \frac{W}{\Delta t} = \frac{dW}{dt}$$

where we have represented the infinitesimal value of the work done by dW. We find from Equation 7.3 that $dW = \mathbf{F} \cdot d\mathbf{r}$. Therefore, the instantaneous power can be written

Instantaneous power

$$\mathcal{P} = \frac{dW}{dt} = \mathbf{F} \cdot \frac{d\mathbf{r}}{dt} = \mathbf{F} \cdot \mathbf{v} \tag{7.23}$$

where we use the fact that $\mathbf{v} = d\mathbf{r}/dt$.

In general, power is defined for any type of energy transfer. Therefore, the most general expression for power is

$$\mathcal{P} = \frac{dE}{dt} \tag{7.24}$$

where dE/dt is the rate at which energy is crossing the boundary of the system by a given transfer mechanism.

The SI unit of power is joules per second (J/s), also called the **watt** (W) (after James Watt):

The watt

$$1 \text{ W} = 1 \text{ J/s} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^3$$

A unit of power in the U.S. customary system is the **horsepower** (hp):

$$1 \text{ hp} = 746 \text{ W}$$

A unit of energy (or work) can now be defined in terms of the unit of power. One **kilowatt-hour** (kWh) is the energy transferred in 1 h at the constant rate of 1 kW = 1 000 J/s. The amount of energy represented by 1 kWh is

$$1 \text{ kWh} = (10^3 \text{ W})(3\,600 \text{ s}) = 3.60 \times 10^6 \text{ J}$$

Note that a kilowatt-hour is a unit of energy, not power. When you pay your electric bill, you are buying energy, and the amount of energy transferred by electrical transmission into a home during the period represented by the electric bill is usually expressed in kilowatt-hours. For example, your bill may state that you used 900 kWh of energy during a month, and you are being charged at the rate of 10¢ per kWh. Your obligation is then $90 for this amount of energy. As another example, suppose an electric bulb is rated at 100 W. In 1.00 hour of operation, it would have energy transferred to it by electrical transmission in the amount of $(0.100 \text{ kW})(1.00 \text{ h}) = 0.100 \text{ kWh} = 3.60 \times 10^5 \text{ J}$.

PITFALL PREVENTION

7.9 W, *W*, and watts

Do not confuse the symbol W for the watt with the italic symbol W for work. Also, remember that the watt already represents a rate of energy transfer, so that "watts per second" does not make sense. The watt is *the same as* a joule per second.

Quick Quiz 7.12 An older model car accelerates from rest to speed v in 10 seconds. A newer, more powerful sports car accelerates from rest to $2v$ in the same time period. What is the ratio of the power of the newer car to that of the older car? (a) 0.25 (b) 0.5 (c) 1 (d) 2 (e) 4

Example 7.12 Power Delivered by an Elevator Motor

An elevator car has a mass of 1 600 kg and is carrying passengers having a combined mass of 200 kg. A constant friction force of 4 000 N retards its motion upward, as shown in Figure 7.19a.

(A) What power delivered by the motor is required to lift the elevator car at a constant speed of 3.00 m/s?

Solution The motor must supply the force of magnitude T that pulls the elevator car upward. The problem states that the speed is constant, which provides the hint that $a = 0$. Therefore we know from Newton's second law that $\Sigma F_y = 0$. The free-body diagram in Figure 7.19b specifies the upward direction as positive. From Newton's second law we obtain

$$\sum F_y = T - f - Mg = 0$$

where M is the *total* mass of the system (car plus passengers), equal to 1 800 kg. Therefore,

$$\begin{aligned} T &= f + Mg \\ &= 4.00 \times 10^3 \text{ N} + (1.80 \times 10^3 \text{ kg})(9.80 \text{ m/s}^2) \\ &= 2.16 \times 10^4 \text{ N} \end{aligned}$$

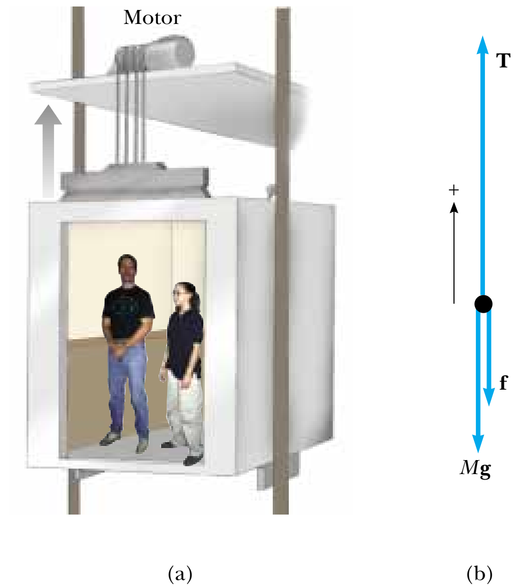


Figure 7.19 (Example 7.12) (a) The motor exerts an upward force $\mathbf{T}$ on the elevator car. The magnitude of this force is the tension T in the cable connecting the car and motor. The downward forces acting on the car are a friction force $\mathbf{f}$ and the gravitational force $\mathbf{F}_g = M\mathbf{g}$. (b) The free-body diagram for the elevator car.

Using Equation 7.23 and the fact that $\mathbf{T}$ is in the same direction as $\mathbf{v}$, we find that

$$\mathcal{P} = \mathbf{T}\cdot\mathbf{v} = Tv$$
$$= (2.16 \times 10^4\ \text{N})(3.00\ \text{m/s}) = \boxed{6.48 \times 10^4\ \text{W}}$$

(B) What power must the motor deliver at the instant the speed of the elevator is v if the motor is designed to provide the elevator car with an upward acceleration of 1.00 m/s^2?

Solution We expect to obtain a value greater than we did in part (A), where the speed was constant, because the motor must now perform the additional task of accelerating the car. The only change in the setup of the problem is that in this case, $a > 0$. Applying Newton's second law to the car gives

$$\sum F_y = T - f - Mg = Ma$$
$$T = M(a + g) + f$$
$$= (1.80 \times 10^3\ \text{kg})(1.00\ \text{m/s}^2 + 9.80\ \text{m/s}^2)$$
$$+ 4.00 \times 10^3\ \text{N}$$
$$= 2.34 \times 10^4\ \text{N}$$

Therefore, using Equation 7.23, we obtain for the required power

$$\mathcal{P} = Tv = \boxed{(2.34 \times 10^4\ \text{N})v}$$

where v is the instantaneous speed of the car in meters per second. To compare to part (A), let $v = 3.00$ m/s, giving a power of

$$\mathcal{P} = (2.34 \times 10^4\ \text{N})(3.00\ \text{m/s}) = 7.02 \times 10^4\ \text{W}$$

This is larger than the power found in part (A), as we expect.

7.9 Energy and the Automobile

Automobiles powered by gasoline engines are very inefficient machines. Even under ideal conditions, less than 15% of the chemical energy in the fuel is used to power the vehicle. The situation is much worse than this under stop-and-go driving conditions in a city. In this section, we use the concepts of energy, power, and friction to analyze automobile fuel consumption.

Many mechanisms contribute to energy loss in an automobile. About 67% of the energy available from the fuel is lost in the engine. This energy ends up in the atmosphere, partly via the exhaust system and partly via the cooling system. (As explained in Chapter 22, energy loss from the exhaust and cooling systems is required by a fundamental law of thermodynamics.) Approximately 10% of the available energy is lost to friction in the transmission, drive shaft, wheel and axle bearings, and differential. Friction in other moving parts transforms approximately 6% of the energy to internal energy, and 4% of the energy is used to operate fuel and oil pumps and such accessories as power steering and air conditioning. This leaves a mere 13% of the available energy to propel the automobile! This energy is used mainly to balance the energy loss due to flexing of the tires and the friction caused by the air, which is more commonly referred to as *air resistance.*

Let us examine the power required to provide a force in the forward direction that balances the combination of the two friction forces. The coefficient of rolling friction μ between the tires and the road is about 0.016. For a 1 450-kg car, the weight is 14 200 N and on a horizontal roadway the force of rolling friction has a magnitude of $\mu n = \mu mg = 227$ N. As the car's speed increases, a small reduction in the normal force

Table 7.2

Friction Forces and Power Requirements for a Typical Car[a]

v(mi/h)	v(m/s)	n(N)	f_r(N)	f_a(N)	f_t(N)	$\mathcal{P} = f_t v$(kW)
0	0	14 200	227	0	227	0
20	8.9	14 100	226	48	274	2.4
40	17.9	13 900	222	192	414	7.4
60	26.8	13 600	218	431	649	17.4
80	35.8	13 200	211	767	978	35.0
100	44.7	12 600	202	1 199	1 400	62.6

[a] In this table, n is the normal force, f_r is rolling friction, f_a is air friction, f_t is total friction, and $\mathcal{P}$ is the power delivered to the wheels.

occurs as a result of decreased pressure as air flows over the top of the car. (This phenomenon is discussed in Chapter 14.) This reduction in the normal force causes a reduction in the force of rolling friction f_r with increasing speed, as the data in Table 7.2 indicate.

Now let us consider the effect of the resistive force that results from the movement of air past the car. For large objects, the resistive force f_a associated with air friction is proportional to the square of the speed (see Section 6.4) and is given by Equation 6.6:

$$f_a = \tfrac{1}{2} D\rho A v^2$$

where D is the drag coefficient, ρ is the density of air, and A is the cross-sectional area of the moving object. We can use this expression to calculate the f_a values in Table 7.2, using $D = 0.50$, $\rho = 1.20 \text{ kg/m}^3$, and $A \approx 2 \text{ m}^2$.

The magnitude of the total friction force f_t is the sum of the rolling friction force and the air resistive force:

$$f_t = f_r + f_a$$

At low speeds, rolling friction is the predominant resistive force, but at high speeds air drag predominates, as shown in Table 7.2. Rolling friction can be decreased by a reduction in tire flexing (for example, by an increase in the air pressure slightly above recommended values) and by the use of radial tires. Air drag can be reduced through the use of a smaller cross-sectional area and by streamlining the car. Although driving a car with the windows open increases air drag and thus results in a 3% decrease in mileage, driving with the windows closed and the air conditioner running results in a 12% decrease in mileage.

The total power needed to maintain a constant speed v is $f_t v$, and this is the power that must be delivered to the wheels. For example, from Table 7.2 we see that at $v = 26.8$ m/s (60 mi/h) the required power is

$$\mathcal{P} = f_t v = (649 \text{ N})(26.8 \text{ m/s}) = 17.4 \text{ kW}$$

This power can be broken down into two parts: (1) the power $f_r v$ needed to compensate for rolling friction, and (2) the power $f_a v$ needed to compensate for air drag. At $v = 26.8$ m/s, we obtain the values

$$\mathcal{P}_r = f_r v = (218 \text{ N})(26.8 \text{ m/s}) = 5.84 \text{ kW}$$

$$\mathcal{P}_a = f_a v = (431 \text{ N})(26.8 \text{ m/s}) = 11.6 \text{ kW}$$

Note that $\mathcal{P} = \mathcal{P}_r + \mathcal{P}_a$ and 67% of the power is used to compensate for air drag.

On the other hand, at $v = 44.7$ m/s(100 mi/h), $\mathcal{P}_r = 9.03$ kW, $\mathcal{P}_a = 53.6$ kW, $\mathcal{P} = 62.6$ kW and 86% of the power is associated with air drag. This shows the importance of air drag at high speeds.

Example 7.13 Gas Consumed by a Compact Car

A compact car has a mass of 800 kg, and its efficiency is rated at 18%. (That is, 18% of the available fuel energy is delivered to the wheels.) Find the amount of gasoline used to accelerate the car from rest to 27 m/s (60 mi/h). Use the fact that the energy equivalent of 1 gal of gasoline is 1.3×10^8 J.

Solution The energy required to accelerate the car from rest to a speed v is equal to its final kinetic energy, $\frac{1}{2}mv^2$:

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(800 \text{ kg})(27 \text{ m/s})^2 = 2.9 \times 10^5 \text{ J}$$

If the engine were 100% efficient, each gallon of gasoline would supply 1.3×10^8 J of energy. Because the engine is only 18% efficient, each gallon delivers an energy of only $(0.18)(1.3 \times 10^8 \text{ J}) = 2.3 \times 10^7$ J. Hence, the number of gallons used to accelerate the car is

$$\text{Number of gal} = \frac{2.9 \times 10^5 \text{ J}}{2.3 \times 10^7 \text{ J/gal}} = \boxed{0.013 \text{ gal}}$$

Let us estimate that it takes 10 s to achieve the indicated speed. The distance traveled during this acceleration is

$$\Delta x = \bar{v}\Delta t = \frac{v_{xf} + v_{xi}}{2}(\Delta t) = \frac{27 \text{ m/s} + 0}{2}(10 \text{ s})$$
$$= 135 \text{ m} \approx 0.08 \text{ mi}$$

At a constant cruising speed, 0.013 gal of gasoline is sufficient to propel the car nearly 0.5 mi, over six times farther. This demonstrates the extreme energy requirements of stop-and-start driving.

Example 7.14 Power Delivered to the Wheels

Suppose the compact car in Example 7.13 has a gas mileage of 35 mi/gal at 60 mi/h. How much power is delivered to the wheels?

Solution We find the rate of gasoline consumption by dividing the speed by the gas mileage:

$$\frac{60 \text{ mi/h}}{35 \text{ mi/gal}} = 1.7 \text{ gal/h}$$

Using the fact that each gallon is equivalent to 1.3×10^8 J, we find that the total power used is

$$\mathcal{P} = (1.7 \text{ gal/h})(1.3 \times 10^8 \text{ J/gal})\left(\frac{1 \text{ h}}{3.6 \times 10^3 \text{ s}}\right)$$
$$= 62 \text{ kW}$$

Because 18% of the available power is used to propel the car, the power delivered to the wheels is $(0.18)(62 \text{ kW}) = \boxed{11 \text{ kW.}}$ This is 37% less than the 17.4-kW value obtained for the 1 450-kg car discussed in the text. Vehicle mass is clearly an important factor in power-loss mechanisms.

Example 7.15 Car Accelerating Up a Hill

Consider a car of mass m that is accelerating up a hill, as shown in Figure 7.20. An automotive engineer measures the magnitude of the total resistive force to be

$$f_t = (218 + 0.70v^2) \text{ N}$$

where v is the speed in meters per second. Determine the power the engine must deliver to the wheels as a function of speed.

Solution The forces on the car are shown in Figure 7.20, in which **F** is the force of friction from the road that propels the car; the remaining forces have their usual meaning.

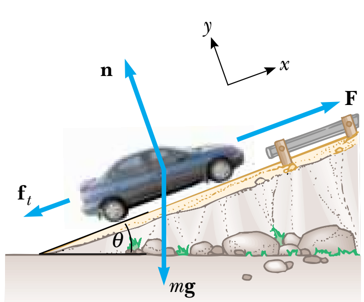


Figure 7.20 (Example 7.15) A car climbs a hill.

Applying Newton's second law to the motion along the road surface, we find that

$$\sum F_x = F - f_t - mg \sin\theta = ma$$
$$F = ma + mg\sin\theta + f_t$$
$$= ma + mg\sin\theta + (218 + 0.70v^2)$$

Therefore, the power required to move the car forward is

$$\mathcal{P} = Fv = mva + mvg\sin\theta + 218v + 0.70v^3$$

The term mva represents the power that the engine must deliver to accelerate the car. If the car moves at constant speed, this term is zero and the total power requirement is reduced. The term $mvg \sin\theta$ is the power required to provide a force to balance a component of the gravitational force as the car moves up the incline. This term would be zero for motion on a horizontal surface. The term $218v$ is the power required to provide a force to balance rolling friction, and the term $0.70v^3$ is the power needed against air drag.

If we take $m = 1\,450$ kg, $v = 27$ m/s (= 60 mi/h), $a = 1.0$ m/s^2, and $\theta = 10°$, then the various terms in $\mathcal{P}$ are calculated to be

$$mva = (1\,450 \text{ kg})(27 \text{ m/s})(1.0 \text{ m/s}^2)$$
$$= 39 \text{ kW} = 52 \text{ hp}$$

$$mvg \sin\theta = (1\,450 \text{ kg})(27 \text{ m/s})(9.80 \text{ m/s}^2)(\sin 10°)$$
$$= 67 \text{ kW} = 89 \text{ hp}$$
$$218v = 218(27 \text{ m/s}) = 5.9 \text{ kW} = 7.9 \text{ hp}$$
$$0.70v^3 = 0.70(27 \text{ m/s})^3 = 14 \text{ kW} = 18 \text{ hp}$$

Hence, the total power required is 126 kW or 167 hp.

Note that the power requirements for traveling at constant speed on a horizontal surface are only 20 kW, or 27 hp (the sum of the last two terms). Furthermore, if the mass were halved (as in the case of a compact car), then the power required also is reduced by almost the same factor.

SUMMARY

Take a practice test for this chapter by clicking on the Practice Test link at http://www.pse6.com.

A **system** is most often a single particle, a collection of particles or a region of space. A **system boundary** separates the system from the **environment**. Many physics problems can be solved by considering the interaction of a system with its environment.

The **work** W done on a system by an agent exerting a constant force $\mathbf{F}$ on the system is the product of the magnitude Δr of the displacement of the point of application of the force and the component $F\cos\theta$ of the force along the direction of the displacement $\Delta\mathbf{r}$:

$$W \equiv F\,\Delta r \cos\theta \tag{7.1}$$

The **scalar product** (dot product) of two vectors $\mathbf{A}$ and $\mathbf{B}$ is defined by the relationship

$$\mathbf{A}\cdot\mathbf{B} \equiv AB\cos\theta \tag{7.2}$$

where the result is a scalar quantity and θ is the angle between the two vectors. The scalar product obeys the commutative and distributive laws.

If a varying force does work on a particle as the particle moves along the x axis from x_i to x_f, the work done by the force on the particle is given by

$$W \equiv \int_{x_i}^{x_f} F_x\,dx \tag{7.7}$$

where F_x is the component of force in the x direction.

The **kinetic energy** of a particle of mass m moving with a speed v is

$$K \equiv \tfrac{1}{2}mv^2 \tag{7.15}$$

The **work–kinetic energy theorem** states that if work is done on a system by external forces and the only change in the system is in its speed, then

$$\sum W = K_f - K_i = \tfrac{1}{2}mv_f^{\,2} - \tfrac{1}{2}mv_i^{\,2} \tag{7.14, 7.16}$$

For a nonisolated system, we can equate the change in the total energy stored in the system to the sum of all the transfers of energy across the system boundary. For an isolated system, the total energy is constant—this is a statement of **conservation of energy.**

If a friction force acts, the kinetic energy of the system is reduced and the appropriate equation to be applied is

$$\Delta K = -f_k d + \sum W_{\text{other forces}} \tag{7.21a}$$

or

$$K_f = K_i - f_k d + \sum W_{\text{other forces}} \tag{7.21b}$$

The **instantaneous power** $\mathcal{P}$ is defined as the time rate of energy transfer. If an agent applies a force $\mathbf{F}$ to an object moving with a velocity $\mathbf{v}$, the power delivered by that agent is

$$\mathcal{P} \equiv \frac{dW}{dt} = \mathbf{F}\cdot\mathbf{v} \tag{7.23}$$

QUESTIONS

1. When a particle rotates in a circle, a force acts on it directed toward the center of rotation. Why is it that this force does no work on the particle?
2. Discuss whether any work is being done by each of the following agents and, if so, whether the work is positive or negative: (a) a chicken scratching the ground, (b) a person studying, (c) a crane lifting a bucket of concrete, (d) the gravitational force on the bucket in part (c), (e) the leg muscles of a person in the act of sitting down.
3. When a punter kicks a football, is he doing any work on the ball while his toe is in contact with it? Is he doing any work on the ball after it loses contact with his toe? Are any forces doing work on the ball while it is in flight?
4. Cite two examples in which a force is exerted on an object without doing any work on the object.
5. As a simple pendulum swings back and forth, the forces acting on the suspended object are the gravitational force, the tension in the supporting cord, and air resistance. (a) Which of these forces, if any, does no work on the pendulum? (b) Which of these forces does negative work at all times during its motion? (c) Describe the work done by the gravitational force while the pendulum is swinging.
6. If the dot product of two vectors is positive, does this imply that the vectors must have positive rectangular components?
7. For what values of θ is the scalar product (a) positive and (b) negative?
8. As the load on a vertically hanging spiral spring is increased, one would not expect the F_s-versus-x graph line to remain straight, as shown in Figure 7.10d. Explain qualitatively what you would expect for the shape of this graph as the load on the spring is increased.
9. A certain uniform spring has spring constant k. Now the spring is cut in half. What is the relationship between k and the spring constant k' of each resulting smaller spring? Explain your reasoning.
10. Can kinetic energy be negative? Explain.
11. Discuss the work done by a pitcher throwing a baseball. What is the approximate distance through which the force acts as the ball is thrown?
12. One bullet has twice the mass of a second bullet. If both are fired so that they have the same speed, which has more kinetic energy? What is the ratio of the kinetic energies of the two bullets?
13. Two sharpshooters fire 0.30-caliber rifles using identical shells. A force exerted by expanding gases in the barrels accelerates the bullets. The barrel of rifle A is 2.00 cm longer than the barrel of rifle B. Which rifle will have the higher muzzle speed?
14. (a) If the speed of a particle is doubled, what happens to its kinetic energy? (b) What can be said about the speed of a particle if the net work done on it is zero?
15. A car salesman claims that a souped-up 300-hp engine is a necessary option in a compact car, in place of the conventional 130-hp engine. Suppose you intend to drive the car within speed limits (≤ 65 mi/h) on flat terrain. How would you counter this sales pitch?
16. Can the average power over a time interval ever be equal to the instantaneous power at an instant within the interval? Explain.
17. In Example 7.15, does the required power increase or decrease as the force of friction is reduced?
18. The kinetic energy of an object depends on the frame of reference in which its motion is measured. Give an example to illustrate this point.
19. Words given precise definitions in physics are sometimes used in popular literature in interesting ways. For example, a rock falling from the top of a cliff is said to be "gathering force as it falls to the beach below." What does the phrase "gathering force" mean, and can you repair this phrase?
20. In most circumstances, the normal force acting on an object and the force of static friction do zero work on the object. However, the reason that the work is zero is different for the two cases. Explain why each does zero work.
21. "A level air track can do no work." Argue for or against this statement.
22. Who first stated the work–kinetic energy theorem? Who showed that it is useful for solving many practical problems? Do some research to answer these questions.

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging ☐ = full solution available in the *Student Solutions Manual and Study Guide*

= coached solution with hints available at http://www.pse6.com = computer useful in solving problem

= paired numerical and symbolic problems

Section 7.2 Work Done by a Constant Force

1. A block of mass 2.50 kg is pushed 2.20 m along a frictionless horizontal table by a constant 16.0-N force directed 25.0° below the horizontal. Determine the work done on the block by (a) the applied force, (b) the normal force exerted by the table, and (c) the gravitational force. (d) Determine the total work done on the block.
2. A shopper in a supermarket pushes a cart with a force of 35.0 N directed at an angle of 25.0° downward from the horizontal. Find the work done by the shopper on the cart as he moves down an aisle 50.0 m long.
3. Batman, whose mass is 80.0 kg, is dangling on the free end of a 12.0-m rope, the other end of which is fixed to a tree limb above. He is able to get the rope in motion

as only Batman knows how, eventually getting it to swing enough that he can reach a ledge when the rope makes a 60.0° angle with the vertical. How much work was done by the gravitational force on Batman in this maneuver?

4. A raindrop of mass 3.35×10^{-5} kg falls vertically at constant speed under the influence of gravity and air resistance. Model the drop as a particle. As it falls 100 m, what is the work done on the raindrop (a) by the gravitational force and (b) by air resistance?

Section 7.3 The Scalar Product of Two Vectors

5. Vector **A** has a magnitude of 5.00 units, and **B** has a magnitude of 9.00 units. The two vectors make an angle of 50.0° with each other. Find $\mathbf{A} \cdot \mathbf{B}$.

6. For any two vectors **A** and **B**, show that $\mathbf{A} \cdot \mathbf{B} = A_xB_x + A_yB_y + A_zB_z$. (*Suggestion:* Write **A** and **B** in unit vector form and use Equations 7.4 and 7.5.)

Note: In Problems 7 through 10, calculate numerical answers to three significant figures as usual.

7. A force $\mathbf{F} = (6\hat{\mathbf{i}} - 2\hat{\mathbf{j}})$ N acts on a particle that undergoes a displacement $\Delta\mathbf{r} = (3\hat{\mathbf{i}} + \hat{\mathbf{j}})$ m. Find (a) the work done by the force on the particle and (b) the angle between **F** and $\Delta\mathbf{r}$.

8. Find the scalar product of the vectors in Figure P7.8.

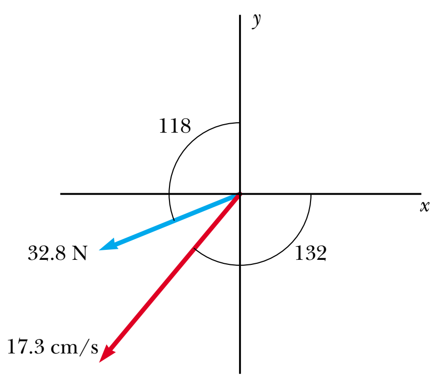


Figure P7.8

9. Using the definition of the scalar product, find the angles between (a) $\mathbf{A} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$ and $\mathbf{B} = 4\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$; (b) $\mathbf{A} = -2\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$ and $\mathbf{B} = 3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$; (c) $\mathbf{A} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\mathbf{B} = 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$.

10. For $\mathbf{A} = 3\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$, $\mathbf{B} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$, and $\mathbf{C} = 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$, find $\mathbf{C} \cdot (\mathbf{A} - \mathbf{B})$.

Section 7.4 Work Done by a Varying Force

11. The force acting on a particle varies as in Figure P7.11. Find the work done by the force on the particle as it moves (a) from $x = 0$ to $x = 8.00$ m, (b) from $x = 8.00$ m to $x = 10.0$ m, and (c) from $x = 0$ to $x = 10.0$ m.

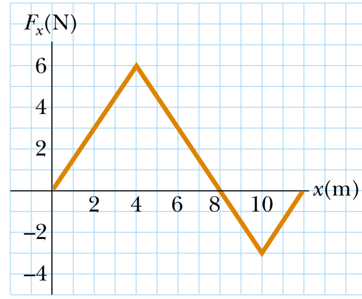


Figure P7.11

12. The force acting on a particle is $F_x = (8x - 16)$ N, where x is in meters. (a) Make a plot of this force versus x from $x = 0$ to $x = 3.00$ m. (b) From your graph, find the net work done by this force on the particle as it moves from $x = 0$ to $x = 3.00$ m.

13. A particle is subject to a force F_x that varies with position as in Figure P7.13. Find the work done by the force on the particle as it moves (a) from $x = 0$ to $x = 5.00$ m, (b) from $x = 5.00$ m to $x = 10.0$ m, and (c) from $x = 10.0$ m to $x = 15.0$ m. (d) What is the total work done by the force over the distance $x = 0$ to $x = 15.0$ m?

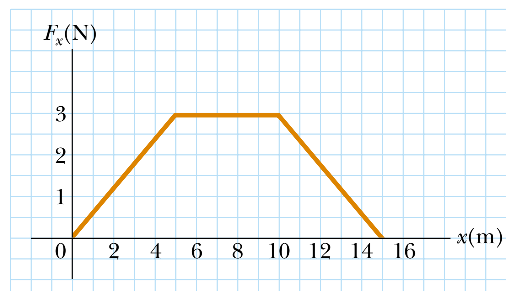


Figure P7.13 Problems 13 and 28.

14. A force $\mathbf{F} = (4x\hat{\mathbf{i}} + 3y\hat{\mathbf{j}})$ N acts on an object as the object moves in the x direction from the origin to $x = 5.00$ m. Find the work $W = \int \mathbf{F} \cdot d\mathbf{r}$ done on the object by the force.

15. When a 4.00-kg object is hung vertically on a certain light spring that obeys Hooke's law, the spring stretches 2.50 cm. If the 4.00-kg object is removed, (a) how far will the spring stretch if a 1.50-kg block is hung on it, and (b) how much work must an external agent do to stretch the same spring 4.00 cm from its unstretched position?

16. An archer pulls her bowstring back 0.400 m by exerting a force that increases uniformly from zero to 230 N. (a) What is the equivalent spring constant of the bow? (b) How much work does the archer do in pulling the bow?

17. Truck suspensions often have "helper springs" that engage at high loads. One such arrangement is a leaf spring with a helper coil spring mounted on the axle, as in Figure P7.17. The helper spring engages when the main leaf spring is compressed by distance y_0, and then helps to support any additional load. Consider a leaf spring constant of 5.25×10^5 N/m, helper spring constant of 3.60×10^5 N/m, and $y_0 = 0.500$ m. (a) What is the

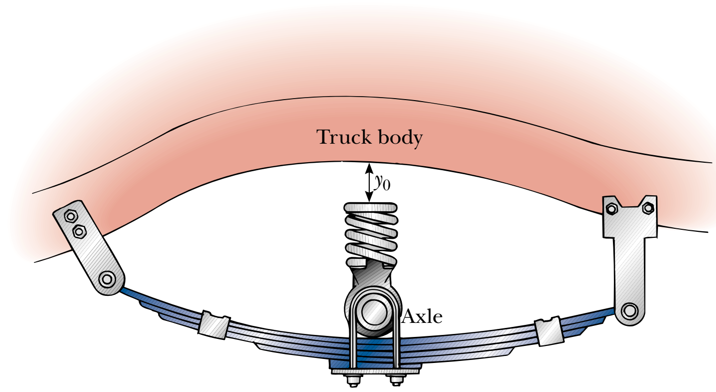


Figure P7.17

compression of the leaf spring for a load of 5.00×10^5 N? (b) How much work is done in compressing the springs?

18. A 100-g bullet is fired from a rifle having a barrel 0.600 m long. Assuming the origin is placed where the bullet begins to move, the force (in newtons) exerted by the expanding gas on the bullet is $15\,000 + 10\,000x - 25\,000x^2$, where x is in meters. (a) Determine the work done by the gas on the bullet as the bullet travels the length of the barrel. (b) **What If?** If the barrel is 1.00 m long, how much work is done, and how does this value compare to the work calculated in (a)?

19. If it takes 4.00 J of work to stretch a Hooke's-law spring 10.0 cm from its unstressed length, determine the extra work required to stretch it an additional 10.0 cm.

20. A small particle of mass m is pulled to the top of a frictionless half-cylinder (of radius R) by a cord that passes over the top of the cylinder, as illustrated in Figure P7.20. (a) If the particle moves at a constant speed, show that $F = mg\cos\theta$. (*Note*: If the particle moves at constant speed, the component of its acceleration tangent to the cylinder must be zero at all times.) (b) By directly integrating $W = \int \mathbf{F} \cdot d\mathbf{r}$, find the work done in moving the particle at constant speed from the bottom to the top of the half-cylinder.

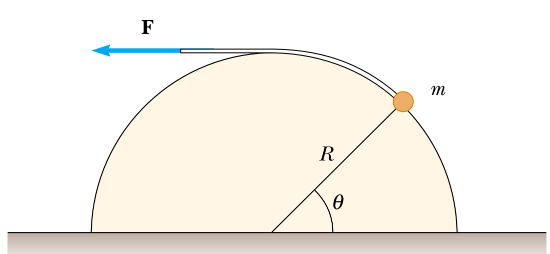


Figure P7.20

21. A light spring with spring constant 1 200 N/m is hung from an elevated support. From its lower end a second light spring is hung, which has spring constant 1 800 N/m. An object of mass 1.50 kg is hung at rest from the lower end of the second spring. (a) Find the total extension distance of the pair of springs. (b) Find the effective spring constant of the pair of springs as a system. We describe these springs as *in series*.

22. A light spring with spring constant k_1 is hung from an elevated support. From its lower end a second light spring is hung, which has spring constant k_2. An object of mass m is hung at rest from the lower end of the second spring. (a) Find the total extension distance of the pair of springs. (b) Find the effective spring constant of the pair of springs as a system. We describe these springs as *in series*.

23. Express the units of the force constant of a spring in SI base units.

Section 7.5 Kinetic Energy and the Work–Kinetic Energy Theorem

Section 7.6 The Nonisolated System—Conservation of Energy

24. A 0.600-kg particle has a speed of 2.00 m/s at point Ⓐ and kinetic energy of 7.50 J at point Ⓑ. What is (a) its kinetic energy at Ⓐ? (b) its speed at Ⓑ? (c) the total work done on the particle as it moves from Ⓐ to Ⓑ?

25. A 0.300-kg ball has a speed of 15.0 m/s. (a) What is its kinetic energy? (b) **What If?** If its speed were doubled, what would be its kinetic energy?

26. A 3.00-kg object has a velocity $(6.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}})$ m/s. (a) What is its kinetic energy at this time? (b) Find the total work done on the object if its velocity changes to $(8.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}})$ m/s. (*Note:* From the definition of the dot product, $v^2 = \mathbf{v} \cdot \mathbf{v}$.)

27. A 2 100-kg pile driver is used to drive a steel I-beam into the ground. The pile driver falls 5.00 m before coming into contact with the top of the beam, and it drives the beam 12.0 cm farther into the ground before coming to rest. Using energy considerations, calculate the average force the beam exerts on the pile driver while the pile driver is brought to rest.

28. A 4.00-kg particle is subject to a total force that varies with position as shown in Figure P7.13. The particle starts from rest at $x = 0$. What is its speed at (a) $x = 5.00$ m, (b) $x = 10.0$ m, (c) $x = 15.0$ m?

29. You can think of the work–kinetic energy theorem as a second theory of motion, parallel to Newton's laws in describing how outside influences affect the motion of an object. In this problem, solve parts (a) and (b) separately from parts (c) and (d) to compare the predictions of the two theories. In a rifle barrel, a 15.0-g bullet is accelerated from rest to a speed of 780 m/s. (a) Find the work that is done on the bullet. (b) If the rifle barrel is 72.0 cm long, find the magnitude of the average total force that acted on it, as $F = W/(\Delta r \cos \theta)$. (c) Find the constant acceleration of a bullet that starts from rest and gains a speed of 780 m/s over a distance of 72.0 cm. (d) If the bullet has mass 15.0 g, find the total force that acted on it as $\Sigma F = ma$.

30. In the neck of the picture tube of a certain black-and-white television set, an electron gun contains two charged metallic plates 2.80 cm apart. An electric force accelerates each electron in the beam from rest to 9.60% of the speed of light over this distance. (a) Determine the kinetic energy of the electron as it leaves the electron gun. Electrons carry this energy to a phosphorescent material on the inner surface of the television screen, making it glow. For an electron passing between the plates in the electron gun, determine (b) the magnitude of the constant electric force acting on the electron, (c) the acceleration, and (d) the time of flight.

Section 7.7 Situations Involving Kinetic Friction

31. A 40.0-kg box initially at rest is pushed 5.00 m along a rough, horizontal floor with a constant applied horizontal force of 130 N. If the coefficient of friction between box and floor is 0.300, find (a) the work done by the applied force, (b) the increase in internal energy in the box-floor system due to friction, (c) the work done by the normal force, (d) the work done by the gravitational force, (e) the change in kinetic energy of the box, and (f) the final speed of the box.

32. A 2.00-kg block is attached to a spring of force constant 500 N/m as in Figure 7.10. The block is pulled 5.00 cm to the right of equilibrium and released from rest. Find the speed of the block as it passes through equilibrium if (a) the horizontal surface is frictionless and (b) the coefficient of friction between block and surface is 0.350.

33. A crate of mass 10.0 kg is pulled up a rough incline with an initial speed of 1.50 m/s. The pulling force is 100 N parallel to the incline, which makes an angle of 20.0° with the horizontal. The coefficient of kinetic friction is 0.400, and the crate is pulled 5.00 m. (a) How much work is done by the gravitational force on the crate? (b) Determine the increase in internal energy of the crate–incline system due to friction. (c) How much work is done by the 100-N force on the crate? (d) What is the change in kinetic energy of the crate? (e) What is the speed of the crate after being pulled 5.00 m?

34. A 15.0-kg block is dragged over a rough, horizontal surface by a 70.0-N force acting at 20.0° above the horizontal. The block is displaced 5.00 m, and the coefficient of kinetic friction is 0.300. Find the work done on the block by (a) the 70-N force, (b) the normal force, and (c) the gravitational force. (d) What is the increase in internal energy of the block-surface system due to friction? (e) Find the total change in the block's kinetic energy.

35. A sled of mass m is given a kick on a frozen pond. The kick imparts to it an initial speed of 2.00 m/s. The coefficient of kinetic friction between sled and ice is 0.100. Use energy considerations to find the distance the sled moves before it stops.

Section 7.8 Power

36. The electric motor of a model train accelerates the train from rest to 0.620 m/s in 21.0 ms. The total mass of the train is 875 g. Find the average power delivered to the train during the acceleration.

37. A 700-N Marine in basic training climbs a 10.0-m vertical rope at a constant speed in 8.00 s. What is his power output?

38. Make an order-of-magnitude estimate of the power a car engine contributes to speeding the car up to highway speed. For concreteness, consider your own car if you use one. In your solution state the physical quantities you take as data and the values you measure or estimate for them. The mass of the vehicle is given in the owner's manual. If you do not wish to estimate for a car, consider a bus or truck that you specify.

39. A skier of mass 70.0 kg is pulled up a slope by a motor-driven cable. (a) How much work is required to pull him a distance of 60.0 m up a 30.0° slope (assumed frictionless) at a constant speed of 2.00 m/s? (b) A motor of what power is required to perform this task?

40. A 650-kg elevator starts from rest. It moves upward for 3.00 s with constant acceleration until it reaches its cruising speed of 1.75 m/s. (a) What is the average power of the elevator motor during this period? (b) How does this power compare with the motor power when the elevator moves at its cruising speed?

41. An energy-efficient lightbulb, taking in 28.0 W of power, can produce the same level of brightness as a conventional bulb operating at power 100 W. The lifetime of the energy efficient bulb is 10 000 h and its purchase price is \$17.0, whereas the conventional bulb has lifetime 750 h and costs \$0.420 per bulb. Determine the total savings obtained by using one energy-efficient bulb over its lifetime, as opposed to using conventional bulbs over the same time period. Assume an energy cost of \$0.080 0 per kilowatt-hour.

42. Energy is conventionally measured in Calories as well as in joules. One Calorie in nutrition is one kilocalorie, defined as 1 kcal = 4 186 J. Metabolizing one gram of fat can release 9.00 kcal. A student decides to try to lose weight by exercising. She plans to run up and down the stairs in a football stadium as fast as she can and as many times as necessary. Is this in itself a practical way to lose weight? To evaluate the program, suppose she runs up a flight of 80 steps, each 0.150 m high, in 65.0 s. For simplicity, ignore the energy she uses in coming down (which is small). Assume that a typical efficiency for human muscles is 20.0%. This means that when your body converts 100 J from metabolizing fat, 20 J goes into doing mechanical work (here, climbing stairs). The remainder goes into extra internal energy. Assume the student's mass is 50.0 kg. (a) How many times must she run the flight of stairs to lose one pound of fat? (b) What is her average power output, in watts and in horsepower, as she is running up the stairs?

43. For saving energy, bicycling and walking are far more efficient means of transportation than is travel by automobile. For example, when riding at 10.0 mi/h a cyclist uses food energy at a rate of about 400 kcal/h above what he would use if merely sitting still. (In exercise physiology, power is often measured in kcal/h rather than in watts. Here 1 kcal = 1 nutritionist's Calorie = 4 186 J.) Walking at 3.00 mi/h requires about 220 kcal/h. It is interesting to compare these values with the energy consumption required for travel by car. Gasoline yields about 1.30×10^8 J/gal. Find the fuel economy in equivalent miles per gallon for a person (a) walking, and (b) bicycling.

Section 7.9 Energy and the Automobile

44. Suppose the empty car described in Table 7.2 has a fuel economy of 6.40 km/liter (15 mi/gal) when traveling at 26.8 m/s (60 mi/h). Assuming constant efficiency, determine the fuel economy of the car if the total mass of passengers plus driver is 350 kg.

45. A compact car of mass 900 kg has an overall motor efficiency of 15.0%. (That is, 15% of the energy supplied by the fuel is delivered to the wheels of the car.) (a) If burning one gallon of gasoline supplies 1.34×10^8 J of energy, find the amount of gasoline used in accelerating the car from rest to 55.0 mi/h. Here you may ignore the effects of air resistance and rolling friction. (b) How many such accelerations will one gallon provide? (c) The mileage claimed for the car is 38.0 mi/gal at 55 mi/h. What power is delivered to the wheels (to overcome frictional effects) when the car is driven at this speed?

Additional Problems

46. A baseball outfielder throws a 0.150-kg baseball at a speed of 40.0 m/s and an initial angle of 30.0°. What is the kinetic energy of the baseball at the highest point of its trajectory?

47. While running, a person dissipates about 0.600 J of mechanical energy per step per kilogram of body mass. If a 60.0-kg runner dissipates a power of 70.0 W during a race, how fast is the person running? Assume a running step is 1.50 m long.

48. The direction of any vector $\mathbf{A}$ in three-dimensional space can be specified by giving the angles α, β, and γ that the vector makes with the x, y, and z axes, respectively. If $\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$, (a) find expressions for $\cos\alpha$, $\cos\beta$, and $\cos\gamma$ (these are known as *direction cosines*), and (b) show that these angles satisfy the relation $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. (*Hint:* Take the scalar product of $\mathbf{A}$ with $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ separately.)

49. A 4.00-kg particle moves along the x axis. Its position varies with time according to $x = t + 2.0t^3$, where x is in meters and t is in seconds. Find (a) the kinetic energy at any time t, (b) the acceleration of the particle and the force acting on it at time t, (c) the power being delivered to the particle at time t, and (d) the work done on the particle in the interval $t = 0$ to $t = 2.00$ s.

50. The spring constant of an automotive suspension spring increases with increasing load due to a spring coil that is widest at the bottom, smoothly tapering to a smaller diameter near the top. The result is a softer ride on normal road surfaces from the narrower coils, but the car does not bottom out on bumps because when the upper coils collapse, they leave the stiffer coils near the bottom to absorb the load. For a tapered spiral spring that compresses 12.9 cm with a 1 000-N load and 31.5 cm with a 5 000-N load, (a) evaluate the constants a and b in the empirical equation $F = ax^b$ and (b) find the work needed to compress the spring 25.0 cm.

51. A bead at the bottom of a bowl is one example of an object in a stable equilibrium position. When a physical system is displaced by an amount x from stable equilibrium, a restoring force acts on it, tending to return the system to its equilibrium configuration. The magnitude of the restoring force can be a complicated function of x. For example, when an ion in a crystal is displaced from its lattice site, the restoring force may not be a simple function of x. In such cases we can generally imagine the function $F(x)$ to be expressed as a power series in x, as $F(x) = -(k_1x + k_2x^2 + k_3x^3 + \ldots)$. The first term here is just Hooke's law, which describes the force exerted by a simple spring for small displacements. For small excursions from equilibrium we generally neglect the higher order terms, but in some cases it may be desirable to keep the second term as well. If we model the restoring force as $F = -(k_1x + k_2x^2)$, how much work is done in displacing the system from $x = 0$ to $x = x_{max}$ by an applied force $-F$?

52. A traveler at an airport takes an escalator up one floor, as in Figure P7.52. The moving staircase would itself carry him upward with vertical velocity component v between entry and exit points separated by height h. However, while the escalator is moving, the hurried traveler climbs the steps of the escalator at a rate of n steps/s. Assume that the height of each step is h_s. (a) Determine the amount of chemical energy converted into mechanical energy by the traveler's leg muscles during his escalator ride, given that

Ron Chapple/FPG

Figure P7.52

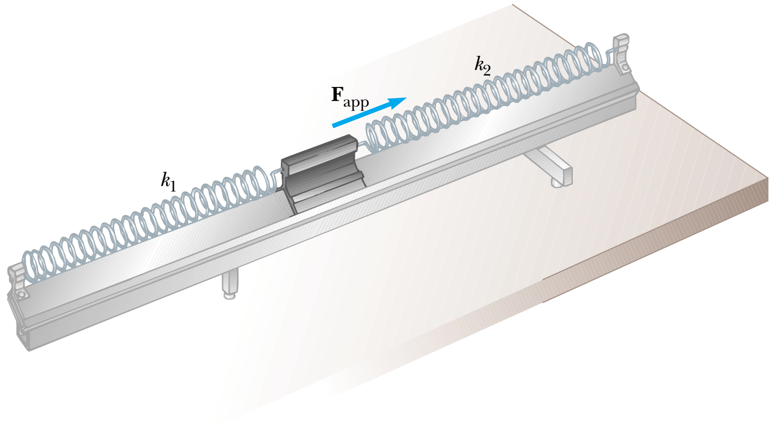


Figure P7.56

his mass is m. (b) Determine the work the escalator motor does on this person.

53. A mechanic pushes a car of mass m, doing work W in making it accelerate from rest. Neglecting friction between car and road, (a) what is the final speed of the car? During this time, the car moves a distance d. (b) What constant horizontal force did the mechanic exert on the car?

54. A 5.00-kg steel ball is dropped onto a copper plate from a height of 10.0 m. If the ball leaves a dent 3.20 mm deep, what is the average force exerted by the plate on the ball during the impact?

55. A single constant force $\mathbf{F}$ acts on a particle of mass m. The particle starts at rest at $t = 0$. (a) Show that the instantaneous power delivered by the force at any time t is $\mathcal{P} = (F^2/m)t$. (b) If $F = 20.0$ N and $m = 5.00$ kg, what is the power delivered at $t = 3.00$ s?

56. Two springs with negligible masses, one with spring constant k_1 and the other with spring constant k_2, are attached to the endstops of a level air track as in Figure P7.56. A glider attached to both springs is located between them. When the glider is in equilibrium, spring 1 is stretched by extension x_{i1} to the right of its unstretched length and spring 2 is stretched by x_{i2} to the left. Now a horizontal force $\mathbf{F}_{\text{app}}$ is applied to the glider to move it a distance x_a to the right from its equilibrium position. Show that in this process (a) the work done on spring 1 is $\frac{1}{2}k_1(x_a^2 + 2x_ax_{i1})$, (b) the work done on spring 2 is $\frac{1}{2}k_2(x_a^2 - 2x_ax_{i2})$, (c) x_{i2} is related to x_{i1} by $x_{i2} = k_1x_{i1}/k_2$, and (d) the total work done by the force F_{app} is $\frac{1}{2}(k_1 + k_2)x_a^2$.

57. As the driver steps on the gas pedal, a car of mass 1 160 kg accelerates from rest. During the first few seconds of motion, the car's acceleration increases with time according to the expression

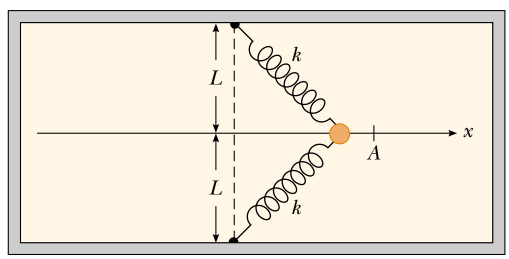
$$a = (1.16\text{ m/s}^3)t - (0.210\text{ m/s}^4)t^2 + (0.240\text{ m/s}^5)t^3$$

(a) What work is done by the wheels on the car during the interval from $t = 0$ to $t = 2.50$ s? (b) What is the output power of the wheels at the instant $t = 2.50$ s?

58. A particle is attached between two identical springs on a horizontal frictionless table. Both springs have spring constant k and are initially unstressed. (a) If the particle is pulled a distance x along a direction perpendicular to the initial configuration of the springs, as in Figure P7.58, show that the force exerted by the springs on the particle is

$$\mathbf{F} = -2kx\left(1 - \frac{L}{\sqrt{x^2 + L^2}}\right)\hat{\mathbf{i}}$$

(b) Determine the amount of work done by this force in moving the particle from $x = A$ to $x = 0$.



Top view

Figure P7.58

59. A rocket body of mass M will fall out of the sky with terminal speed v_T after its fuel is used up. What power output must the rocket engine produce if the rocket is to fly (a) at its terminal speed straight up; (b) at three times the terminal speed straight down? In both cases assume that the mass of the fuel and oxidizer remaining in the rocket is negligible compared to M. Assume that the force of air resistance is proportional to the square of the rocket's speed.

60. **Review problem.** Two constant forces act on a 5.00-kg object moving in the xy plane, as shown in Figure P7.60. Force $\mathbf{F}_1$ is 25.0 N at 35.0°, while $\mathbf{F}_2$ is 42.0 N at 150°. At time $t = 0$, the object is at the origin and has velocity $(4.00\hat{\mathbf{i}} + 2.50\hat{\mathbf{j}})$ m/s. (a) Express the two forces in unit-vector notation. Use unit-vector notation for your other answers. (b) Find the total force on the object. (c) Find the object's acceleration. Now, considering the instant $t = 3.00$ s, (d) find the object's velocity, (e) its location, (f) its kinetic energy from $\frac{1}{2}mv_f^2$, and (g) its kinetic energy from $\frac{1}{2}mv_i^2 + \Sigma\mathbf{F}\cdot\Delta\mathbf{r}$.

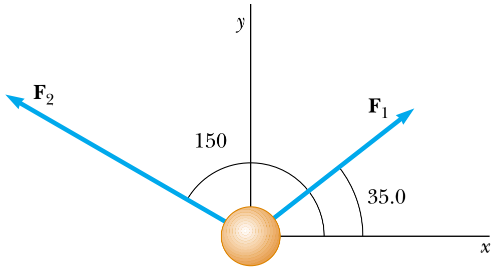


Figure P7.60

61. A 200-g block is pressed against a spring of force constant 1.40 kN/m until the block compresses the spring 10.0 cm. The spring rests at the bottom of a ramp inclined at 60.0° to the horizontal. Using energy considerations, determine how far up the incline the block moves before it stops (a) if there is no friction between the block and the ramp and (b) if the coefficient of kinetic friction is 0.400.

62. When different weights are hung on a spring, the spring stretches to different lengths as shown in the following table. (a) Make a graph of the applied force versus the extension of the spring. By least-squares fitting, determine the straight line that best fits the data. (You may not want to use all the data points.) (b) From the slope of the best-fit line, find the spring constant k. (c) If the spring is extended to 105 mm, what force does it exert on the suspended weight?

F (N)	2.0	4.0	6.0	8.0	10	12	14	16	18	20	22
L (mm)	15	32	49	64	79	98	112	126	149	175	190

63. The ball launcher in a pinball machine has a spring that has a force constant of 1.20 N/cm (Fig. P7.63). The surface on which the ball moves is inclined 10.0° with respect to the horizontal. If the spring is initially compressed 5.00 cm, find the launching speed of a 100-g ball when the plunger is released. Friction and the mass of the plunger are negligible.

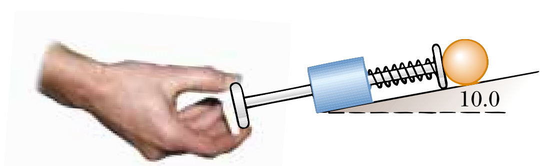


Figure P7.63

64. A 0.400-kg particle slides around a horizontal track. The track has a smooth vertical outer wall forming a circle with a radius of 1.50 m. The particle is given an initial speed of 8.00 m/s. After one revolution, its speed has dropped to 6.00 m/s because of friction with the rough floor of the track. (a) Find the energy converted from mechanical to internal in the system due to friction in one revolution. (b) Calculate the coefficient of kinetic friction. (c) What is the total number of revolutions the particle makes before stopping?

65. In diatomic molecules, the constituent atoms exert attractive forces on each other at large distances and repulsive forces at short distances. For many molecules, the Lennard-Jones law is a good approximation to the magnitude of these forces:

$$F = F_0\left[2\left(\frac{\sigma}{r}\right)^{13} - \left(\frac{\sigma}{r}\right)^{7}\right]$$

where r is the center-to-center distance between the atoms in the molecule, σ is a length parameter, and F_0 is the force when $r = \sigma$. For an oxygen molecule, we find that $F_0 = 9.60 \times 10^{-11}$ N and $\sigma = 3.50 \times 10^{-10}$ m. Determine the work done by this force if the atoms are pulled apart from $r = 4.00 \times 10^{-10}$ m to $r = 9.00 \times 10^{-10}$ m.

66. As it plows a parking lot, a snowplow pushes an ever-growing pile of snow in front of it. Suppose a car moving through the air is similarly modeled as a cylinder pushing a growing plug of air in front of it. The originally stationary air is set into motion at the constant speed v of the cylinder, as in Figure P7.66. In a time interval Δt, a new disk of air of mass Δm must be moved a distance $v\Delta t$ and hence must be given a kinetic energy $\frac{1}{2}(\Delta m)v^2$. Using this model, show that the automobile's power loss due to air resistance is $\frac{1}{2}\rho A v^3$ and that the resistive force acting on the car is $\frac{1}{2}\rho A v^2$, where ρ is the density of air. Compare this result with the empirical expression $\frac{1}{2}D\rho A v^2$ for the resistive force.

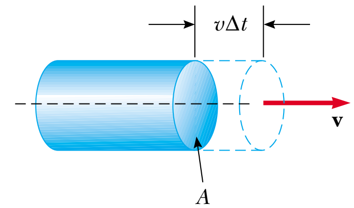


Figure P7.66

67. A particle moves along the x axis from $x = 12.8$ m to $x = 23.7$ m under the influence of a force

$$F = \frac{375}{x^3 + 3.75x}$$

where F is in newtons and x is in meters. Using numerical integration, determine the total work done by this force on the particle during this displacement. Your result should be accurate to within 2%.

68. A windmill, such as that in the opening photograph of this chapter, turns in response to a force of high-speed air resistance, $R = \frac{1}{2}D\rho A v^2$. The power available is $\mathcal{P} = Rv = \frac{1}{2}D\rho\pi r^2 v^3$, where v is the wind speed and we have assumed a circular face for the windmill, of radius r. Take the drag coefficient as $D = 1.00$ and the density of air from the front endpaper. For a home windmill with $r = 1.50$ m, calculate the power available if (a) $v = 8.00$ m/s and (b) $v = 24.0$ m/s. The power delivered to the generator is limited by the efficiency of the system, which is about 25%. For comparison, a typical home needs about 3 kW of electric power.

69. More than 2 300 years ago the Greek teacher Aristotle wrote the first book called *Physics*. Put into more precise terminology, this passage is from the end of its Section Eta:

Let $\mathcal{P}$ be the power of an agent causing motion; w, the thing moved; d, the distance covered; and Δt, the time interval required. Then (1) a power equal to $\mathcal{P}$ will in a period of time equal to Δt move $w/2$ a distance $2d$; or (2) it will move $w/2$ the given distance d in the time interval $\Delta t/2$. Also, if (3) the given power $\mathcal{P}$ moves the given object w a distance $d/2$ in time interval $\Delta t/2$, then (4) $\mathcal{P}/2$ will move $w/2$ the given distance d in the given time interval Δt.

(a) Show that Aristotle's proportions are included in the equation $\mathcal{P}\Delta t = bwd$ where b is a proportionality constant. (b) Show that our theory of motion includes this part of Aristotle's theory as one special case. In particular, describe a situation in which it is true, derive the equation representing Aristotle's proportions, and determine the proportionality constant.

70. Consider the block-spring-surface system in part (b) of Example 7.11. (a) At what position x of the block is its speed a maximum? (b) In the **What If?** section of this example, we explored the effects of an increased friction force of 10.0 N. At what position of the block does its maximum speed occur in this situation?

Answers to Quick Quizzes

7.1 (a). The force does no work on the Earth because the force is pointed toward the center of the circle and is therefore perpendicular to the direction of the displacement.

7.2 c, a, d, b. The work in (c) is positive and of the largest possible value because the angle between the force and the displacement is zero. The work done in (a) is zero because the force is perpendicular to the displacement. In (d) and (b), negative work is done by the applied force because in neither case is there a component of the force in the direction of the displacement. Situation (b) is the most negative value because the angle between the force and the displacement is 180°.

7.3 (d). Answer (a) is incorrect because the scalar product $(-\mathbf{A}) \cdot (-\mathbf{B})$ is equal to $\mathbf{A} \cdot \mathbf{B}$. Answer (b) is incorrect because $AB \cos(\theta + 180°)$ gives the negative of the correct value.

7.4 (d). Because of the range of values of the cosine function, $\mathbf{A} \cdot \mathbf{B}$ has values that range from AB to $-AB$.

7.5 (a). Because the work done in compressing a spring is proportional to the square of the compression distance x, doubling the value of x causes the work to increase fourfold.

7.6 (b). Because the work is proportional to the square of the compression distance x and the kinetic energy is proportional to the square of the speed v, doubling the compression distance doubles the speed.

7.7 (a) For the television set, energy enters by electrical transmission (through the power cord) and electromagnetic radiation (the television signal). Energy leaves by heat (from hot surfaces into the air), mechanical waves (sound from the speaker), and electromagnetic radiation (from the screen). (b) For the gasoline-powered lawn mower, energy enters by matter transfer (gasoline). Energy leaves by work (on the blades of grass), mechanical waves (sound), and heat (from hot surfaces into the air). (c) For the hand-cranked pencil sharpener, energy enters by work (from your hand turning the crank). Energy leaves by work (done on the pencil) and mechanical waves (sound).

7.8 (b). The friction force represents an interaction with the environment of the block.

7.9 (b). The friction force represents an interaction with the environment of the surface.

7.10 (a). The friction force is internal to the system, so there are no interactions with the environment.

7.11 (c). The brakes and the roadway are warmer, so their internal energy has increased. In addition, the sound of the skid represents transfer of energy away by mechanical waves.

7.12 (e). Because the speed is doubled, the kinetic energy is four times as large. This kinetic energy was attained for the newer car in the same time interval as the smaller kinetic energy for the older car, so the power is four times as large.